Calculations Policy

Version:	1.0
Author:	Headteacher
Approved by:	Advisory Body
Date:	Autumn 2020
Review date:	Autumn 2023

Staploe Education Trust
Excellence through partnership

Contents

Progression towards a standard written method of calculation

Introduction

This calculation policy has been written in line with the programmes of study taken from the revised **National Curriculum for Mathematics (2014).** It provides guidance on appropriate calculation methods and progression. The content is set out in progressive steps under the following headings: addition, subtraction, multiplication and division.

Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an effective written method confidently and accurately.

Aims of the Policy

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations.
- To ensure that children can use these methods accurately with confidence and understanding.

How to Use this Policy

- Use the policy as the basis of planning but ensure you use previous or following steps guidance to allow for personalised learning.
- Always use ‘Assessment for Learning’ to identify suitable next steps in calculation for groups of children to the previous stage in calculation.
- Always use suitable resources, models and images to support children’s understanding of calculation and place value, as appropriate at all stages. All children, regardless of age and ability should make progress through a ‘C.P.A’ approach (Concrete, Pictorial, Abstract).
- Encourage children to make sensible choices about the methods they use when solving problems.

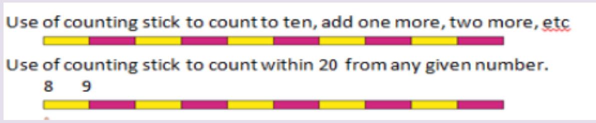
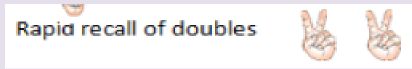
This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Key Language: Addition	total, parts and wholes, plus, add, altogether, more, greater, 'is equal to' 'is the same as', parts, whole, addend, sum					
	SPINE https://www.ncetm.org.uk/resources/50639					
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers. Adding two two-digit numbers.	Column method regrouping. Using place value counters and base 10. (up to 3 digits).	Column method regrouping. Using place value counters and base 10. (up to 3 digits).	Column method regrouping. Use of place value counters and base 10 for adding decimals.	Column method regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Key Language: Subtraction	take away, less than, subtract, minus, fewer, decrease, parts, whole, subtrahend, minuend, difference					
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters and base 10.)	Column method with regrouping. (up to 4 digits using place value counters and base 10.)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.

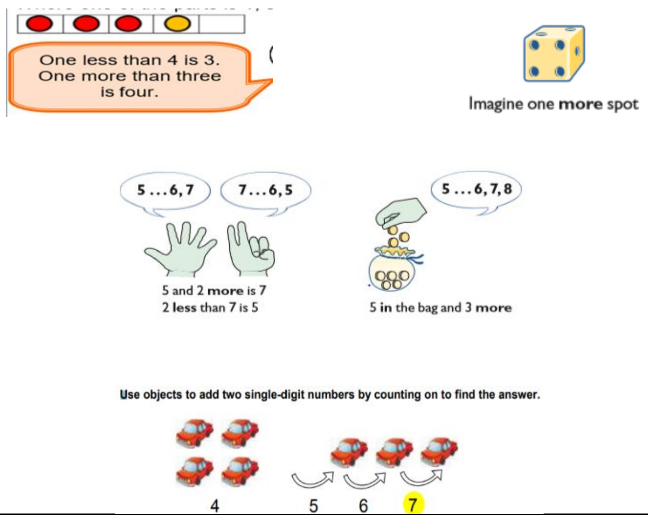
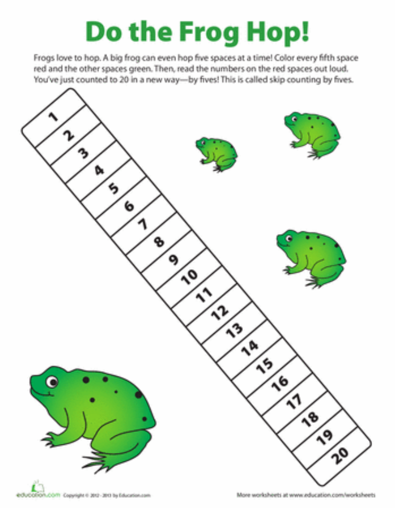
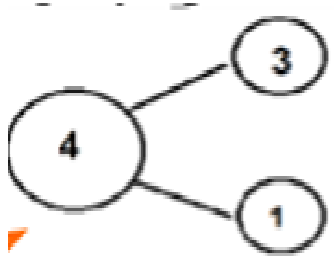
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Key Language: Multiplication	double, times, multiplied by, groups of, lots of, equal groups/parts, multiplier, multiplicand, product					
	SPINE https://www.ncetm.org.uk/resources/50639					
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom.	Arrays- showing commutative multiplication Repeated addition	Arrays Repeated addition 2d × 1d using base 10 Column multiplication- introduced alongside other methods, with place value counters.	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication. Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits).	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Key Language: Division	share, group, divide, divided by, half, equal groups/parts, divisor, dividend, quotient, fraction					

Addition

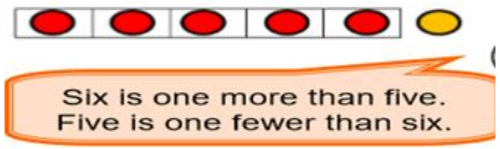
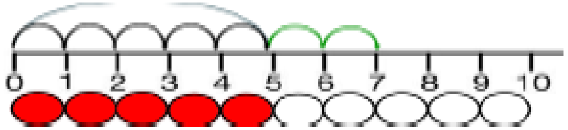
EYFS – Addition (When planning ensure you track forwards to year 1)

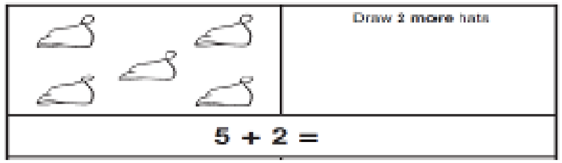
Early Learning Goal 11	Resources
• Children count reliably with numbers from 1 to 20, place them in order • Say which number is one more or one less than a given number • Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.	• https://www.ncetm.org.uk/resources/50724 Calculation strategies within 10 year 1 • https://www.ncetm.org.uk/resources/50719 - Year 1 part, part whole model • NRICH Curriculum Mapping Documents: https://nrich.maths.org/content/id/13291/EYFSKS1CurriculumLinkedtoNRICH.pdf
Potential barriers/misconceptions	**Daily Mental calculation practice to develop fluency in key skills:**
• Unable to recite numbers in the correct order • Not associating number names with objects in group • Unable to count without putting in line or touching • Not being able to 'hold' the number they started with when adding the second group. • Not knowing the number order when counting on from any given number	  • Use of songs/rhymes, marbles, dice games • Use floppy fingers to show pairs to 10. I say 3 children fold 3 fingers down

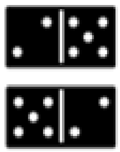
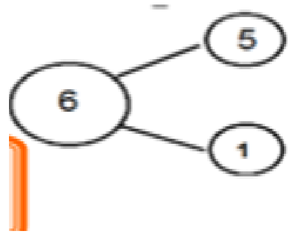
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. Introduce short division, ready for year 4. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td>Counting on</td><td>Where one of the parts is 1; signifying one more or 1 less.

</td><td>

</td><td>

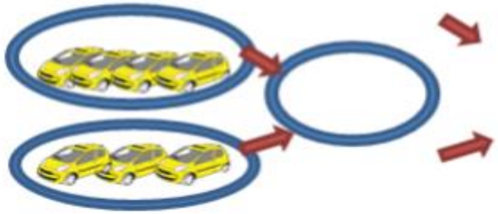
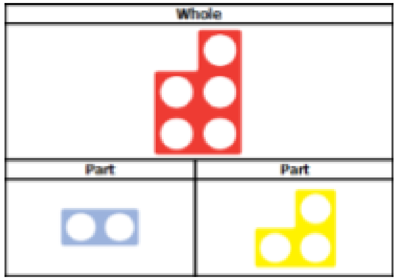
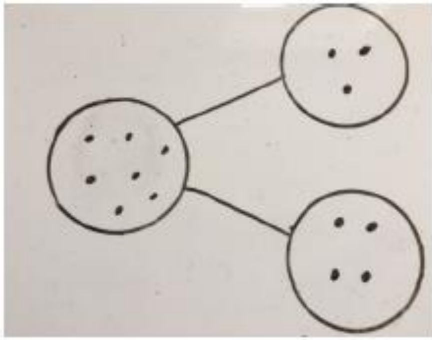
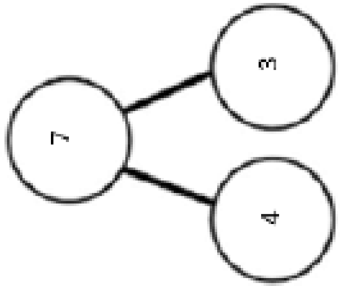
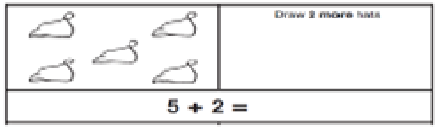
3 + 1 = 4
1 + 3 = 4
4 - 1 = 3
4 - 3 = 1</td></tr>
<tr><td>Count on from 5</td><td>
2 more than 5.

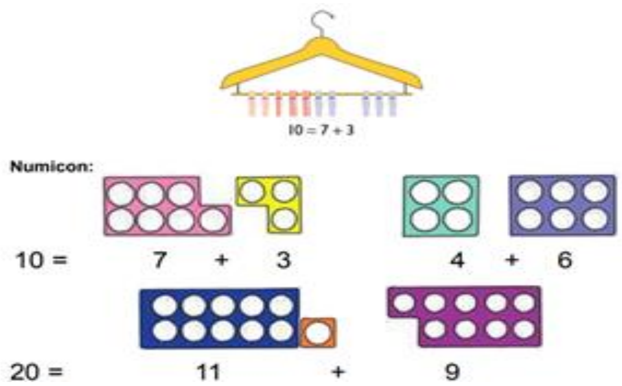
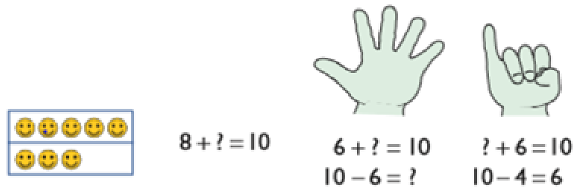
Think 5
Where the whole is 6 or 7 and one of the parts is 5 (to secure benchmark from 5).

</td><td>

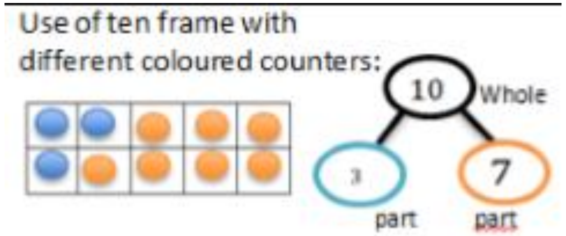
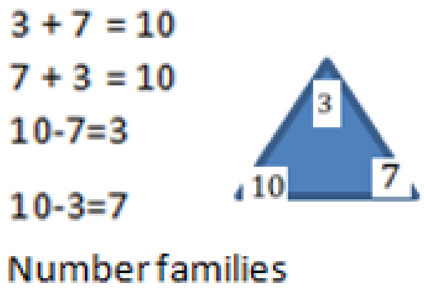



2 + 5 = 7
5 + 2 = 7</td><td>Emphasis should be on the language
‘1 more than 5 is equal to 6.’
‘2 more than 5 is 7.’
‘8 is 3 more than 5.’


6 = 5 + 1
6 = 1 + 5
6 - 1 = 5
6 - 5 = 1</td></tr>
</table>

Combining two parts to make a whole.	 Three paper plates can be used to represent part, part whole. Children move the cars together into one group to find the total amount. (starting with 0-5) 	 **Part, Part, Whole** __ is a part, __ is a part, __ is the whole. Four and three makes seven. Three more than four is seven.	 4+3=7 3+4=7 7-4=3 7-3=4
Bar model	Progression towards a bar model – (or use unifix) 	Sara has 2 apples. Jon has 5 apples. How many apples do they have altogether? How many more apples does Jon have than Sara? 	To read aloud: 2+ 5 = 7 7= 5+□ Knowing that + is add = is equals (the same as) 

<table>
<tr>
<td>Use known number facts – number bonds to 10</td>
<td>Turn the coathanger around to get 3+7 =10 (exposes commutivty)

</td>
<td>


</td>
<td>

To use the inverse
Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting on as their main strategy</td>
</tr>
</table>

Year 1 –Addition (When planning ensure you track back to EYFS and forwards to year 2)

National Curriculum:

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20 including zero
- Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9

Teaching resources

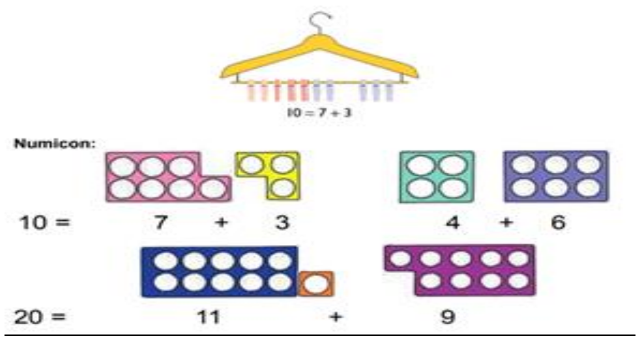
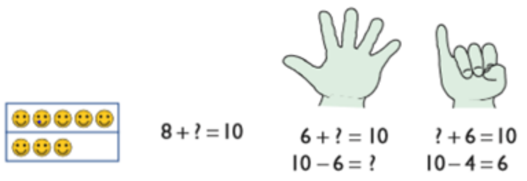
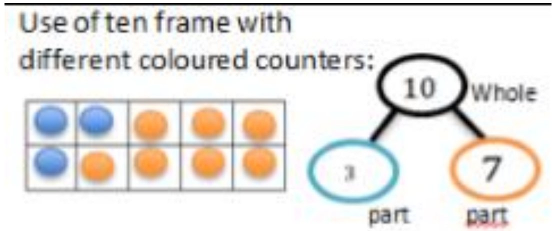
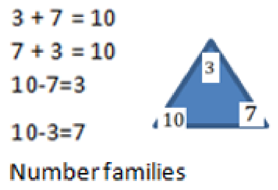
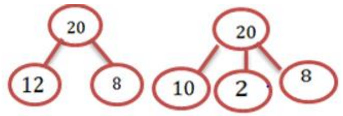
- The national curriculum planning and resource tool years 1-6 https://www.ncetm.org.uk/resources/41211
- Nrich curriculum mapping EYFS – KS1 https://nrich.maths.org/content/id/13291/EYFSKS1CurriculumLinkedtoNRICH.pdf
- The Spine - Resources for addition and subtraction Years 1 to 6 https://www.ncetm.org.uk/resources/50640
- Progression maps with reasoning skills - https://www.ncetm.org.uk/resources/44672

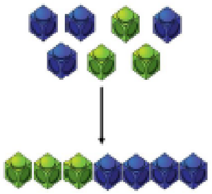
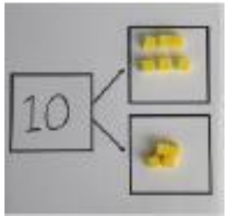
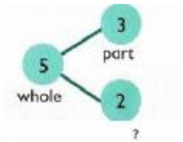
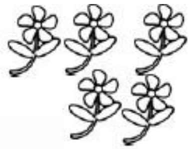
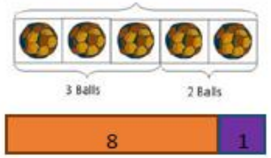
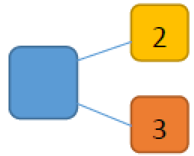
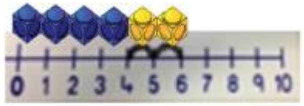
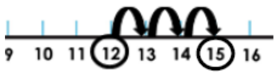
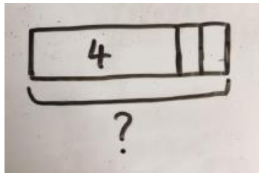
Potential barriers/misconceptions

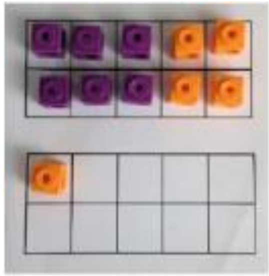
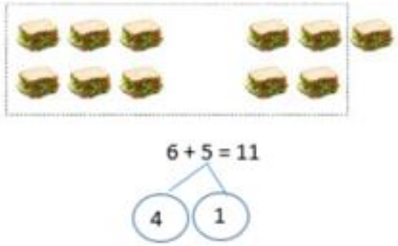
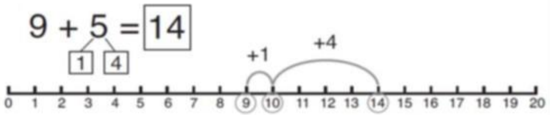
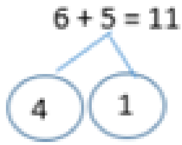
- Learn the pairs of numbers to 10 and 20 but not the pairs that total each number up to 20.
- Don't understand the commutativity of 3+7 = 7+ 3
- Don't associate number facts e.g 13+4=17 and 17-4=13 as they don't see + and – as inverse.
- Only able to complete empty box questions when on right hand side (answer) rather than any position. 3+ □ = 8
- When counting on from a given number, include the start number in their counting. (6+3 = 6,7,8 = 8) rather than 6+3= 7,8,9 = 9)
- Count on and back in 10s and 1s not combining i.e. when add 9, add 10 and subtract 1 (adjust)

Daily Mental calculation practice to develop fluency in key skills:

- To add multiples of ten.
- To add ten to any two-digit number by counting in 10s.
- Bridge through ten (and 20 etc.) when adding a single digit number. (Making ten). i.e. 8 + 6 = 8 + 2 + 4 = 14
- Count on from the largest number
- Rapid recall of number bonds Use of near doubles to add 6 + 7 = 6 + 6+ 1 = 13)
- Add 9 to a single digit number by adding 10 and subtracting 1 (adjust).
- Number bonds ('story of' 5, 6, 7, 8, 9 and 10)

Objective & Strategy	Concrete	Pictorial	Abstract
Use known number facts – number bonds to 10	Turn the coat hanger around to get 3+7 =10 (exposes commutivty) 	 	To use the inverse  Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting on as their main strategy **To add pairs to 20** 

Combining two parts to make a whole: part- whole model	Use part-part whole model. Use cubes to add two numbers together as a group or in a bar (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3 Use the part-part-whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer. A bar model encourages the children to count on, rather than count all.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10. *This is an essential skill for column addition later.*	Start with the bigger number and use the smaller number to make 10. Use ten frames.  Bead strings can also be used to model this. 9+3=9+1+2 	Use pictures or a number line with beadstring. Regroup or partition the smaller number using the part part whole model to make 10.  	 $6 + 4 = 10$ $10 + 1 = 11$ Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

Year 2 –Addition (When planning ensure you track back to Year 1 and forwards to Year 2

National Curriculum:

Solve problems with addition and subtraction:

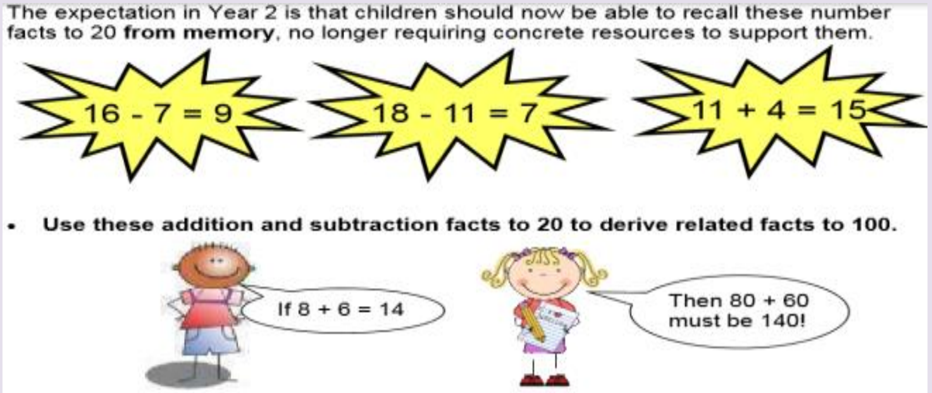
- Using concrete and pictorial representations, including those involving numbers, quantities and measures.
- Applying their increasing knowledge of mental and written methods.
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- A two digit number and ones
- A two digit number and tens
- Add two two-digit numbers
- Adding three one digit numbers
- Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

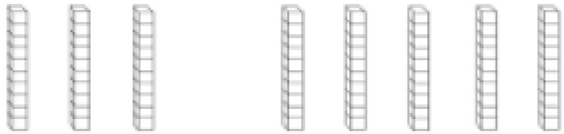
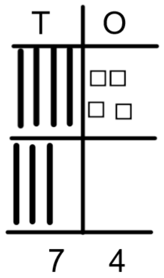
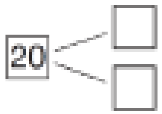
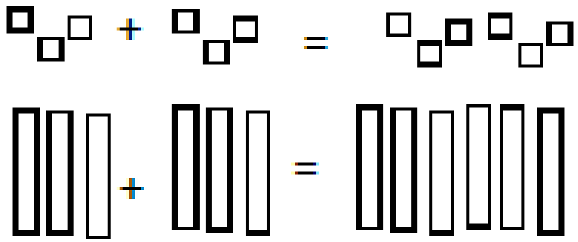
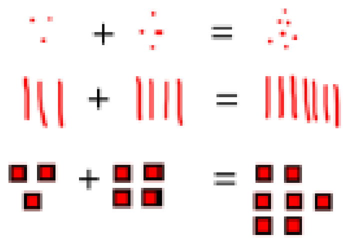
Potential barriers/misconceptions

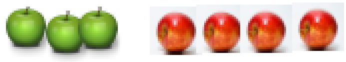
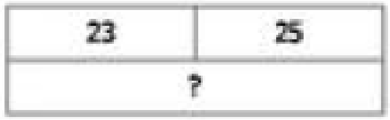
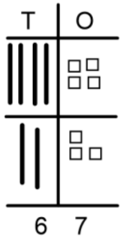
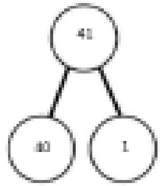
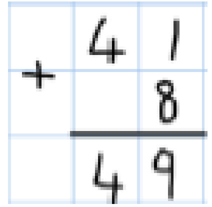
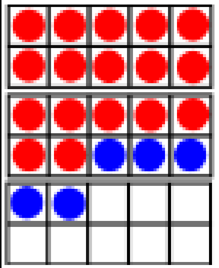
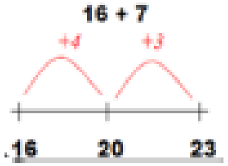
- Pupils believe they have to add in the order that the question was asked (not understanding that addition can be done in any order to do mental calculations more efficiently).
- Pupils still don't have secure rapid recall of addition facts. i.e. struggle to identify all possible missing numbers in _ + _ = 7. (Number bonds).
- Make mistakes counting teen numbers or crossing boundaries.
- Insecure in making links between addition and subtraction and/or recognising inverse.
- In vertical addition- placing the answer in the wrong column. i.e. 24 as 2 in the ones, 4 in tens

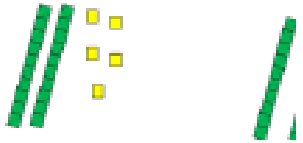
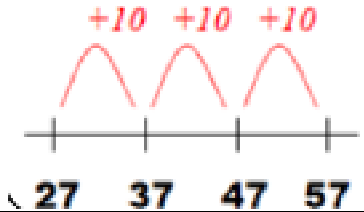
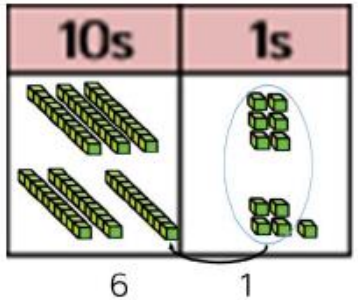
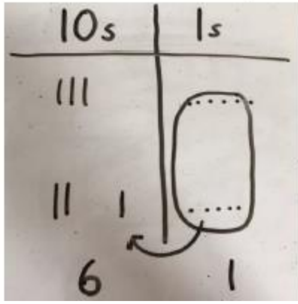
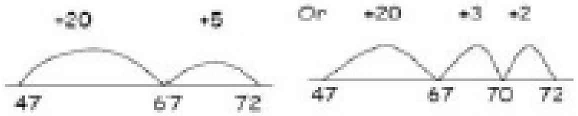
Daily Mental calculation practice to develop fluency in key skills:

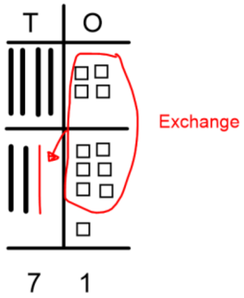
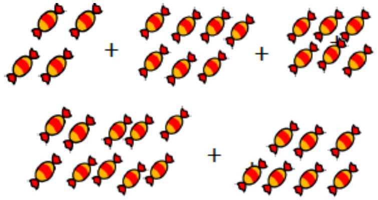
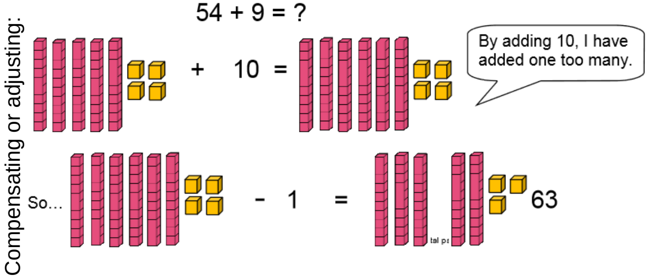
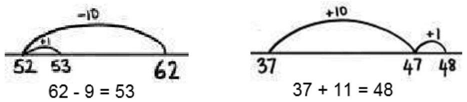
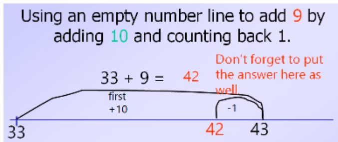
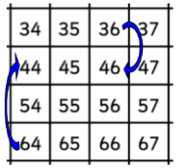
- Mental Maths Counting forwards/ backwards from any given 1 and 2 digit number.
- Rapid recall **of all addition facts to 20 & 100**
- Partitioning adding the ones and then the tens: 24+13 = 4+3+ 20+10=37
- 'Make ten' adding three one digit numbers: 6+7 = 6+4+3= 13 .
- Compensating 24+9= 24+10-1=33 or 42+21= 42+20+1= 63 (adjust).
- Near doubles: 30+29 = double 30-1 and 14+15 is double 14 + 1 or double 15-1.
- Add a single-digit number to any 2-digit number using number facts, including bridging multiples of 10. (E.g. 45 + 4, 38 + 7).
- Add any pair of 2-digit numbers.
- Pupils practice addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3+7=10; 10-7=3 and 7=10-3 to calculate 30+70=100; 100-70=30 and 70=100-30.
- https://www.ncetm.org.uk/resources/50640#yr2



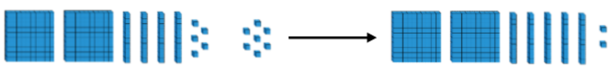
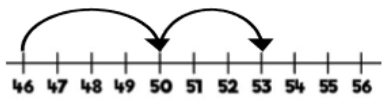
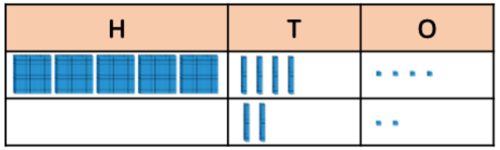
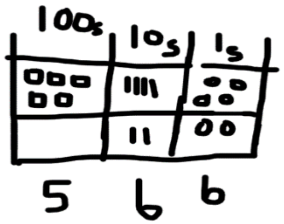
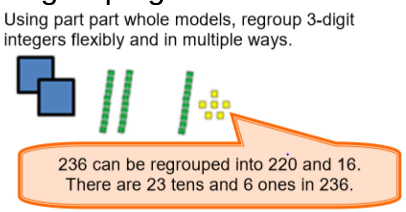
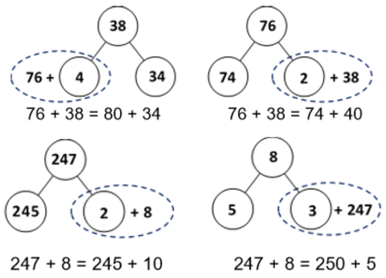
Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	Model using dienes and bead strings 50= 30 = 20 	Use representations for base ten.  3 tens + 5 tens = _____ tens 30 + 50 = _____ 	20 + 30 = 50 70 = 50 + 20 40 + □ = 60 $$\begin{array}{r} 44 \\ +\ 30 \\ \hline 74 \\ \hline \end{array}$$
Use known number facts	Children explore ways of making numbers within 20 Part part whole 	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		Children draw representations of H, T and O 	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

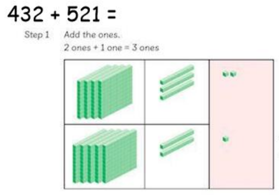
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a 2-digit number and ones	41 + 8 = 	Children to draw a line for 10s and clear circles for ones. Squares can later be made for 100s 	41 + 8  1 + 8 = 9 40 + 9 = 49 
Add a 2-digit number and ones bridging 10	17 + 5 = 22  Use ten frame to make 'magic ten'. Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 17 + 5 = 22  	Rules of equality. Children can use the inverse to solve missing number problems when crossing one number to the other side of the equation. 17+5=22 27+5=□

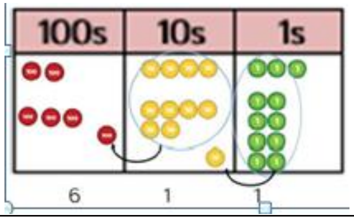
Add a 2-digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	**27 + 30** 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers (partition)	TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	36 + 25= 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: $\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	22+47 = 69 $\begin{array}{r} 47 + \\ \underline{22} \\ \underline{69} \end{array}$

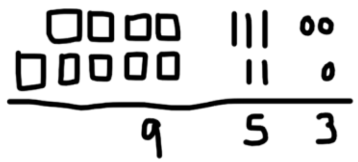
Add two 2-digit numbers crossing 10	 Model using dienes , place value counters and numicon		$$\begin{array}{r} 44 \\ 27 \\ \hline 71 \\ \hline 1 \end{array} +$$
Add three 1-digit numbers using number bonds to bridge 10	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.
Adding 9 and 19 compensating or adjusting	Children can also use this knowledge to help them **add or subtract 9 or 11,** by adding/ subtracting 10 and then **adjusting by 1**. 54 + 9 = ? 	With time and practice, children will be able to use this strategy mentally.  	Once children have grasped this concept using concrete resources, they can move on to using more abstract, pictorial representations.  A **hundred square** is a useful tool with regards to enabling children to add or subtract 10s from any number. It will also reinforce the idea that the **units don't change** but that the **tens increase** or **decrease** respectively. For example: 36 + 10 = 46 Or: 64 - 20 = 44 They can then move left or right with a counter to add or subtact the 1.

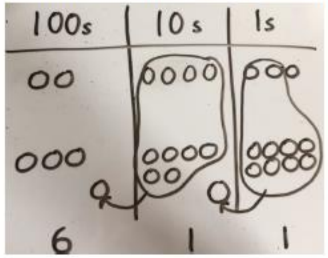
Year 3 -Addition (When planning ensure you track back to Year 2 and forwards to year 4)	
National Curriculum : Add and subtract numbers mentally, including: • A three-digit number and ones • A three- digit number and tens • A three- digit number and hundreds • Add and subtract numbers with up to three digits, using formal written methods of column addition and subtraction • Estimate the answer to a calculation and use inverse operations to check answers. • Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.	**Daily Mental calculation practice to develop fluency in key skills:** • Rapid recall of all addition facts up to and including 20 • Derive quickly addition doubles from 1+1 to 20+20 e.g. 19+19=38 • Doubles of multiples of 5 from 5+5 to 100+100 e.g. 95+95 = 190 • Derive quickly pairs of multiples of 5 that total 100: e.g. 65 + 35 • Know by heart all multiples of 100 that total 1000: e.g. 400 + 600 = 1000 • Add several numbers by making ten & adjusting when adding 11 or 9 add 10 and +1/-1. • Partition and recombine: e.g. 24 + 35 = 20 + 30 + 4 + 5 = 59 • Identify the corresponding subtraction facts. e.g. 22+57 = 79 and 79-57=22 etc. • Add a two-digit number to a multiple of 100.e.g. 200+64 Add a two-digit number to a multiple of 10 crossing 100. e.g. 80 + 34 = 114 • Add 10 to any number crossing the hundreds boundary. e.g. 196 + 10 • Add a pair of multiples of 10, crossing 100. e.g. 90 + □ = 130 • Add pairs of multiples of 100 crossing 1000. e.g. 500 + 800 Add 100 to any 3 digit number, without crossing 1000. e.g. 347 + 100 = □
Potential barriers/misconceptions • Children may still not be secure with all addition facts for each number to 20 • Confused that addition is associative- 3+1 = 4 and 1+3=4 • Find it challenging to mentally add using 'near multiples of 10' • Not sure about which way to compensate: 26+19= 26 +20 – 1 often confused as 36 + 20 + 1. Sometimes begin adding with the left hand column first • Not understanding the concept of regrouping when the number totals more than ten, hundred etc. • Children find it difficult to add when there is a zero involved • Children don't understand importance of zero as a placeholder	**Mental calculations** **True or false?** Are these number sentences true or false? 597 + 7 = 614 804 – 70 = 744, 768 + 140 = 908 Give your reasons. **Hard and easy questions** Which questions are easy / hard? 323 + 10 = 393 + 10 = 454 - 100 = 954 - 120 = Explain why?

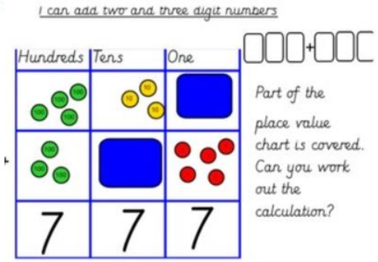
Objective & Strategy	Concrete	Pictorial	Abstract
Adding 3-digit and 1-digit numbers	We can use Base 10 to solve 245 + 7 	We can use a number line to calculate 346 + 7  46 + 7 = 53 so 346 + 7 = 353	346 + 7 = 346 + 4 + 3
Adding 3-digit and 2-digit numbers	544+22  500 + 60 + 6 Links to expanded column method	 	544+22=566
Regrouping	Regrouping is an essential skill  	 	76+38= 247+8 =

<table>
<tr>
<td>Adding 3-digit and 3-digit numbers</td>
<td>Start with base 10 to allow chn to see the size of numbers before moving on to counters.


With exchange: 243 + 368 =
When there are 10 ones in the 1s column we exchange for 1 ten, when there are 10 tens in the 10s column we exchange for 1 hundred.


As children move on to decimals, money and decimal place value counters can be used to support learning.</td>
<td>100s 10s 1s


Chidren to represent the counters in a place value chart, circling when they make an exchange.



</td>
<td>432+521=953

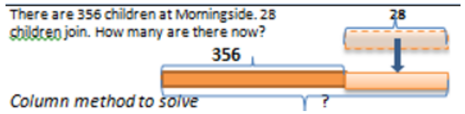
243+368=

First use expanded method

100s 10s 1s
200 40 3
+ 300 60 8
100 10
600 10 1

Followed by compact method:

243
+ 368
11
611


</td>
</tr>
</table>

Year 4 -Addition (When planning ensure you track back to Year 3 and forwards to year 5)

National Curriculum

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two step word problems in context, deciding which operations and methods to use and why.

Potential barriers/misconceptions

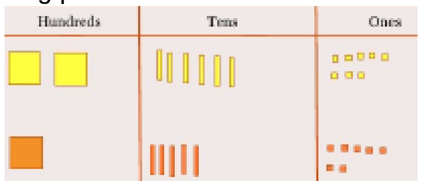
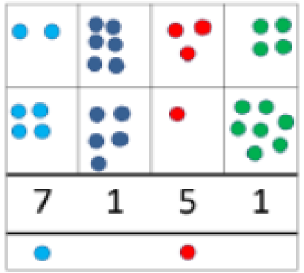
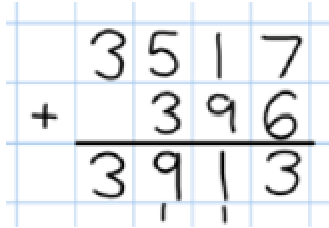
- Children sometimes begin adding with the left hand column first
- Pupils line up numbers from left to right rather than right to left. i.e. 3056 + 254:

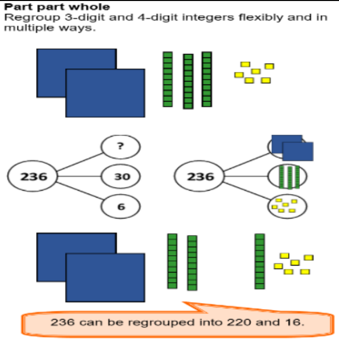
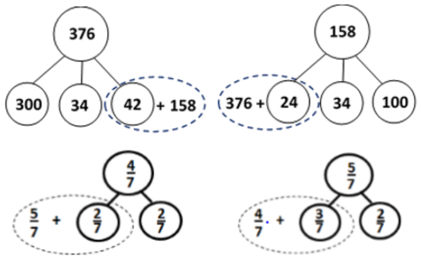
Th	H	T	O
3	0	5	6
2	5	4	

- Not understanding the concept of ‘regroup’ when a number totals more than ten, hundred etc.
- As numbers get larger pupils miscalculate because of lack of understanding of place value
- Some pupils will not remember to add the ten/hundred that they have regrouped
- Pupils don’t use estimation skills to predict answer
- Lack of understanding around value of decimal numbers
- Forgetting to include or line up decimal point

Daily Mental calculation practice to develop fluency in key skills:

- Rapid recall of all addition facts to 20. (e.g. all pairs of numbers to 15)
- Derive quickly related facts: e.g. 9+6=15, 90+60=150, 900+600=1500
- Derive quickly number pairs that make 100. 34 + □ = 100; □ + 45=100
- Derive pairs of multiples of 50 that total 1000: e.g. 250+750
- Derive quickly addition doubles from: 1+1 to 50+50 e.g.
- Double 46 Multiples of 10 from 10+10 to 500+500: e.g. double 280
- Multiples of 100 from 100+100 to 5000+5000: e.g. double 17000
- Count on from any given number in repeated steps of 1,10,100,1000
- Partition into hundreds, tens and ones to add mentally
- Add or subtract the nearest multiple of 10, 100 or 1000 and adjust: add 9, 19, 29 or 11, 21, 31 to any number. e.g. 48+ 61 = 48+60+1
- Identify addition and subtraction facts for any given algorithm.
- Add three numbers mentally. (two-digit and one digit)
- Add three digit multiples of 10: e.g. 430+360 or 570+260
- Find what to add to a three-digit number to make the next higher multiple of 100. e.g. 246+ □ = 300
- Add numbers to 1 decimal place to make the next whole number. 3.4 +□ = 4.0

Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. Move to using place value counters 	Draw representations using pv grid. 	Continue from previous work to carry hundreds as well as tens. 

Regroup numbers in different ways		**Regrouping** This can be adapted to 'Think 100' 376 + 158 	376+158=376+24+34+100 $\frac{4}{7}+\frac{5}{7}=$

Year 5 -Addition (When planning ensure you track back to Year 4 and forwards to year 6)

National Curriculum:

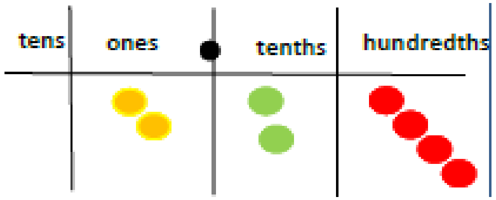
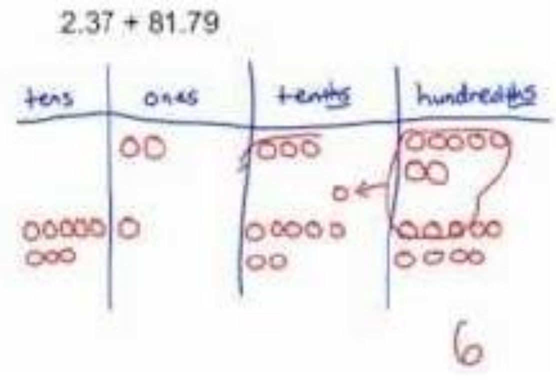
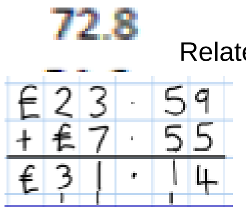
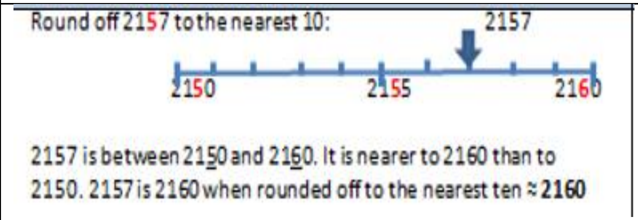
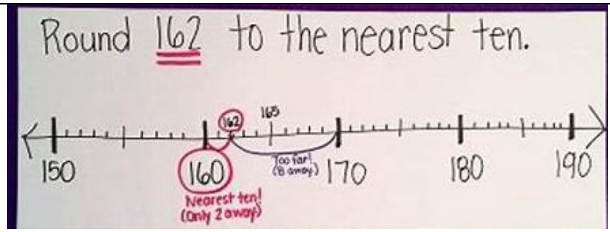
- Add and subtract whole numbers with more than 4 digits, including formal written methods (column addition and subtraction)
- Add and subtract numbers mentally with increasingly large numbers.
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Potential barriers/misconceptions:

- As numbers get larger, pupils miscalculate due to lack of understanding of place value
- Some pupils will not realise that they need to add the regrouped number
- Pupils sometimes forget to line up the decimal points when adding using the column method- particularly when adding mixed amounts. e.g. £4.50 + 72p

Daily Mental calculation practice to develop fluency in key skills:

- Practise mental calculations with increasingly large numbers to aid fluency (e.g. 12,462+ 4200= 16,662)
- Add four-digit multiples of 100 e.g. 3700 + 4500
- Add three or more digit multiples of 100 e.g. 400 + 800 + 500
- Add a single-digit multiple of 100 to a three or four-digit number crossing 1000 e.g. 300 + 876 = □ , 300 + □ = 1176, □ + 876 = 1176 & 38 + 500= □
- Add a three-digit multiple of 10 to a three digit number without crossing the hundreds boundary. e.g. 230+364, 460+518
- Find what to add to a three digit number to make the next higher multiple of 100. E.g. 651 + □ = 700
- Find what to add to a decimal with units and tenths to make the next higher whole number e.g. 8.25 + □ = 9.0.
- Practice estimating to predict and check answers rounding to nearest 10 or 100

Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		 Relate to money and measures. Missing number problems 3 4 □ 2 + 1 3 2 9 4 7 9 1 1 1 1 .□7 + 0 . 9 1 2 .6 8
Rounding to the nearest 10 and 100			To estimate: **1296 + 2508** Children encouraged to articulate their thinking: '3800 is my estimate because I rounded 1296 to 1300 and 2508 to 2500. 1300 +2500 = 3800' (To check I did 1300 + 2508 = 3808 – 4 = 3804)

Year 6 -Addition (When planning ensure you track back to Year 5 for progression)

National Curriculum

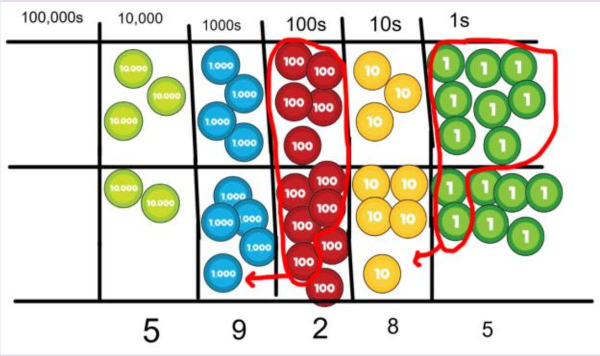
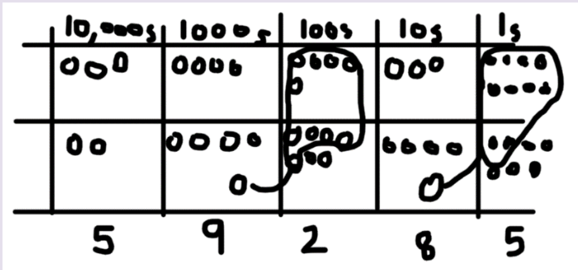
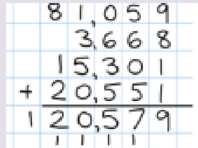
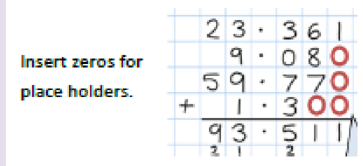
- Perform mental calculations, including with mixed operations and large numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and determine, the in context of a problem, an appropriate degree of accuracy

Potential barriers/misconceptions

- Unless a pupil has a good understanding of place value they will continue to make mistakes with column addition.
- Such errors are often dismissed as common mistakes, when the pupil in fact has a fundamental weakness in their understanding
- When adding the decimals such details are highlighted with the positioning of the decimal point
- Pupils get mixed up with operation and signs when there are subtraction and negative signs in a problem. i.e. important to refer to operation as subtract/ add and these signs as positive/negative

Daily Mental calculation practice to develop fluency in key skills:

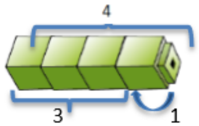
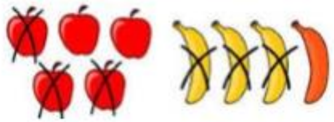
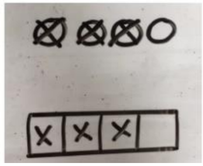
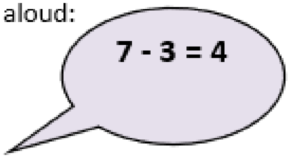
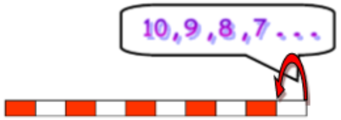
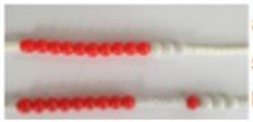
- Find the difference by counting up through the next multiple of 10, 100 or 1000: 7000 - 3675 is +5 + 20 + 300 + 3000= 3325
- Identify near doubles: 421 + 387 = 808 (double 400 plus 21 minus 13)
- Add or subtract the nearest multiple of 10, 100 or 1000 adjust: add 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 etc. by adding 1,2,3 and adjusting by 0.1.
- Add or subtract four digit multiples of 100
- Find what to add to a decimal with units, 10th and 100ths to make the next higher whole number or 10th. What must be added to 7.78 to make 8?
- Add or subtract a pair of decimal fractions each less than 1 and with up to 2 decimal places

Objective & Strategy	Concrete	Pictorial	Abstract
Add several numbers of increasing complexity. Including adding money, measure and decimals with different numbers of decimal points	34, 538 + 24,747 = □ 		  

Subtraction

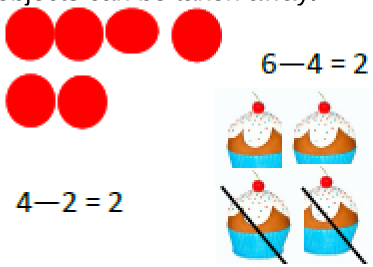
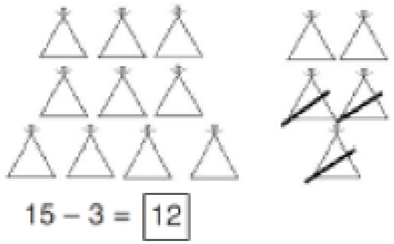
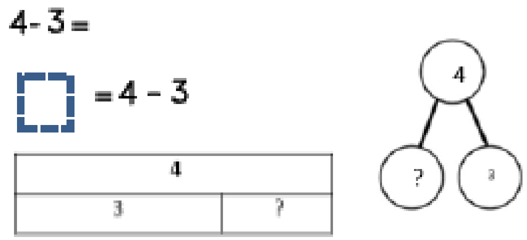
EYFS 1 – Subtraction (When planning ensure you track forwards to EYFS 2 & year 1)

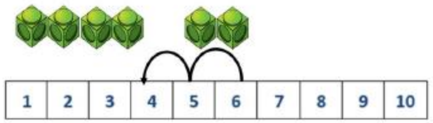
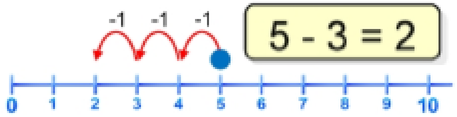
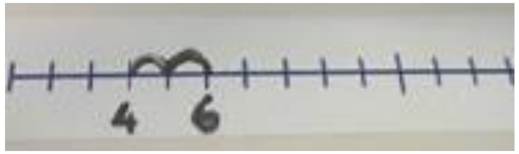
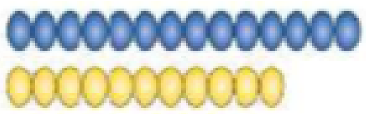
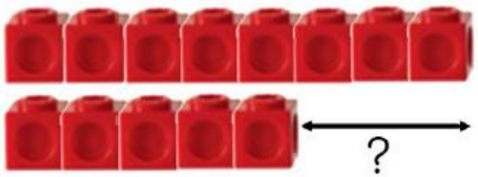
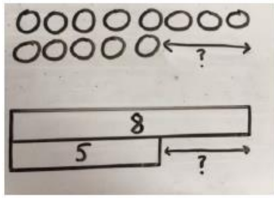
Early Learning Goal 11	Potential barriers/misconceptions	Mental strategies (can revisited throughout day once concept has explicitly shared):
• Children count reliably with numbers from 1 to 20, place them in order (see number and place value) and say which number is one more or one less than a given number • Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.	• Unable to recite numbers in the correct order • Not associating number names with objects in group • Unable to count without putting in line or touching • Not being able to 'hold' the number they started with when taking away from the group • Not knowing the number order when counting backwards.	• Join in rhymes and sing songs such as: Five little ducks went swimming one day; Five little speckled frogs; Five little monkeys jumping on the bed; Five current buns in the baker's shop; Alice the camel has ten humps; Ten green bottles; One man went to mow... • Say the number name that goes before a given number. Count forwards and backwards using a counting stick

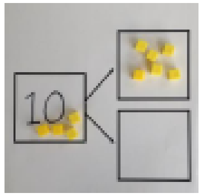
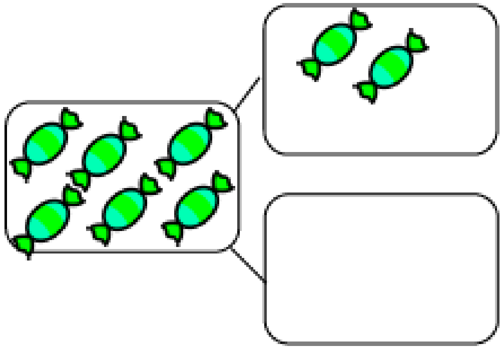
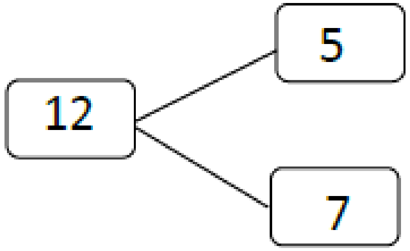
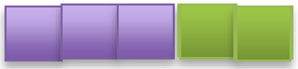
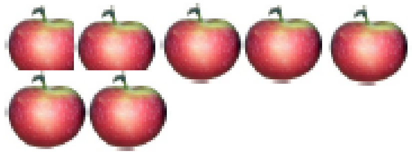
Objective & Strategy	Concrete	Pictorial	Abstract
To recognise 1 less	**Progression to the bar model** **Children start by subtracting objects from a group**  **What is one less than 4?** **Children then use unifix cubes, counting back from the greater number, to find the total number of cubes.**  4 -1 = 3 	 5 – 3 = 2 4 – 3 = 1 Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	To read aloud:  Knowing that - is subtract/ take away = is equals
To count back	  Move the beads along the bead string as you count backwards.	 	7-3=4

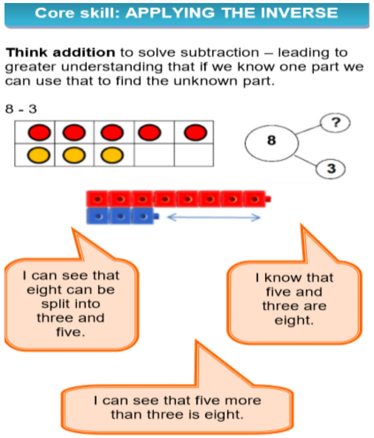
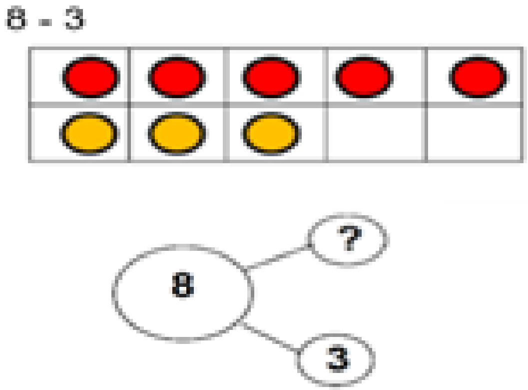
Year 1 – Subtraction (When planning ensure you track forwards to Year 2)

National Curriculum / Potential barriers/misconceptions	Mental strategies
National Curriculum: • Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs • Represent and use number bonds and related subtraction facts within 20 • Add and subtract one-digit and two-digit numbers to 20 including zero • Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9 **Potential barriers/misconceptions:** • Lack of confidence in numbers bonds within ten, to ten and to twenty will prohibit children from fully understanding the rules of commutativity • Children are confident with counting 'up' but have limited experience counting backwards from any given number • Don't associate number facts (e.g. 13+4=17 and 17-4=13) as they don't see + and – as inverse.	**Mental strategies:** • Counting stick: counting forwards and backwards in steps (not only of ones) from any given number 7-3 = □ count back in ones from 7 15-3= count back in ones from 15 18 - 6= count back in twos from 18 • To use 'count back from' strategies. (8-6= 7,6,5,4,3,2… =2) • To use 'count back to' strategies. (8-6 = 7,6 = 2) • Find a small difference by counting up (When two numbers are close together i.e. 15-12=3 counting up from 12 to 15 gives 3.) Subtract ten from a teens number: 19-10= □□□19-□= 9; □-10= 9 • Subtract ten from any two digit number, without crossing: 100: 49-10 = □; 49 - □ = 10 ; □-10 = 39 • Subtract a pair of multiples of ten without crossing 100: 50–20=□ ; 50 - □ = 30 ; □ – 20 = 30

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  	 Cross out drawn objects to show what has been taken away.	

Counting back	 Move the beads along the bead string as you count backwards. **Counting back** (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4  	 Count back in ones using a number line. Next step – empty number line  	*Put 13 in your head, count back 4. What number are you at?* 6 – 4 = □
Find the difference	 The difference between 11 and 14 is 3. Compare objects and amounts: **Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5.  	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  **Count on using a number line to find the difference.**  	*8 is 3 more than 5* *8 - 5=3* *The difference between 8 and 5 is 3* Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference. *Jack is 11 and his sister is 5. How much older is Jack?*

Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Bridging 10	 Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5. 14 – 5	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16-8 = How many ones should we subtract first? (6). How many 1s do we still need to subtract?
Bar model	5 - 2 = 3  Three and two is five  Five subtract three is two  Five subtract two is three	 5—2 = 3	5-2=3 5-3=2 Inverse 3+2 = 5 2+3= 5 (number families) If we know that… then what other facts do we know?

<table>
<tr>
<td>Apply inverse</td>
<td>
</td>
<td>
</td>
<td>8 - __ = 3 =

8 – 3 = __</td>
</tr>
</table>

Year 2 – Subtraction (When planning ensure you track forwards to Year 3)

National Curriculum Solve problems with addition and subtraction:

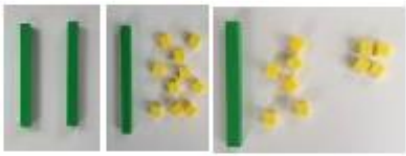
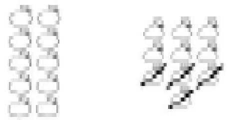
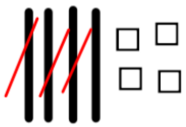
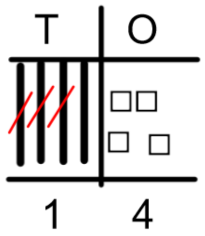
- Using concrete and pictorial representations, including those involving numbers, quantities and measures.
- Applying their increasing knowledge of mental and written methods.
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - A two-digit number and ones
  - A two-digit number and tens
  - Add two two-digit numbers
  - Adding three one digit numbers
- Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

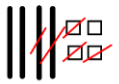
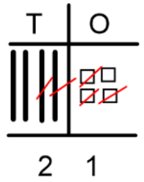
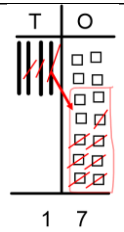
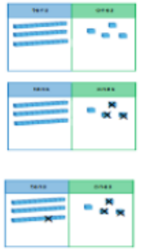
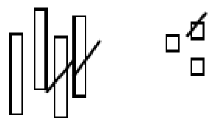
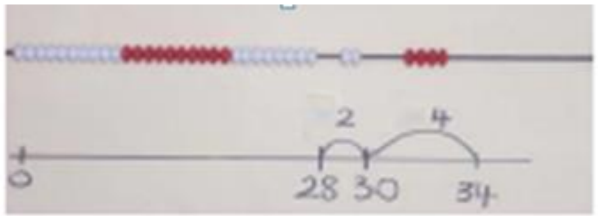
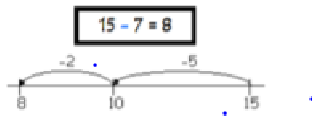
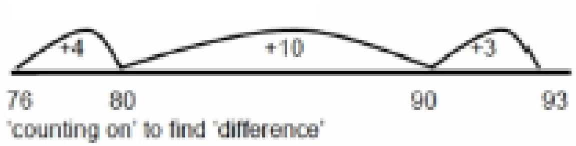
Potential barriers/misconceptions:

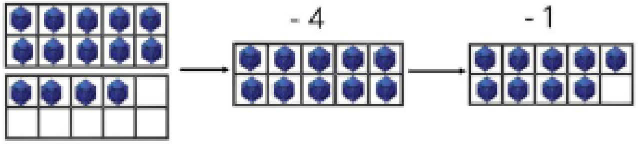
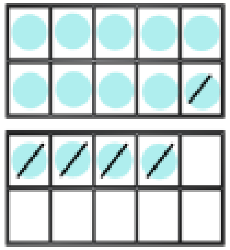
- Avoid telling children 'you can't take a big number away from a smaller number' you can - this will then go into negative numbers. This could lead to misconceptions at a later point.
- Children may not understand the commutative law and believe that it is possible to change any addition and subtraction around.
- Children sometimes regroup but see the new number as one and not ten.
- Pupils may struggle to see 'find the difference' as a form of subtraction.
- This can be linked to lack of consolidated skills in counting on and back.
- Challenge in recalling addition and subtraction facts to 20
- Difficulty using mental strategies to add and subtract two digit numbers

Mental strategies

- To know by heart all addition and subtraction facts for each number to 20
- To use number bonds for mental subtraction. 9-4= □ (Think of addition: 4 and 5 make 9 therefor 9-4=5)
- To subtract multiples of ten from any two-digit number
- To add and subtract mentally a 'near multiple of ten' to or from a two-digit number (15+39 = 1+39+10+4=54)
- To find pairs of numbers with a difference of 10, a difference of 9 etc.
- To find a small difference when counting up; 84-78 = 79,80,81,82,83,84 = 6
- To mentally subtract 11 or 21 or 9 or 19 from any two-digit number; 70-11=59 as it is the same as 70-10-1= 59 24-9=15 because it's the same as 24-10+1 = 15
- Add or subtract any single digit from any two-digit number without crossing the tens boundary (86-□=82)
- Subtract multiples of ten without crossing 100. (90-40=□)
- Subtract multiples of 100 without crossing 1000 (700 – 300 = □)
- Use number bonds to find a small difference between a pair of numbers lying either side of a multiple of 10 (102-97 = 2+3 = 5)

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 20 − 4 =	20—4 = 16
2s-10's	44 - 30 = 		44 30 - 14

2d-2d	44 - 23 = 		44 - 23 21
2d-2d (crossing 10)	44 - 27 = 		$\overset{3}{\cancel{4}}\overset{1}{4}$ - 27 17
Partitioning to subtract without regrouping *'Friendly numbers'*	 34—13 = 21 Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22 40 + 3 - 20 + 1 20 + 2 48 - 7 41
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34 - 6 Count back 4 to bridge 10. Then count back remaining 2 using knowledge of pairs to 10	 Use a number line to count on to next ten and then the rest. 	15-7=8 93—76 = 17

	Making 10 using ten frames. 14 – 5 	 Children to present the ten frame pictorially and discuss what they did to make 10.	$14 - 5 = 9$ 4 1 $14 - 4 = 10$ $10 - 1 = 9$ Children to show how they can make 10 by partitioning the subtrahend.

Year 3 – Subtraction (When planning ensure you track forwards to Year 4)

National Curriculum:

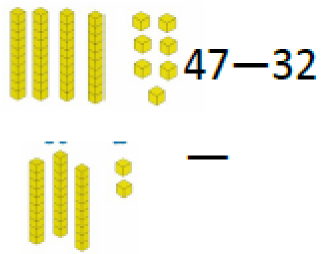
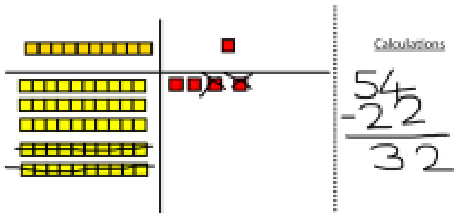
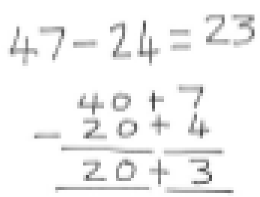
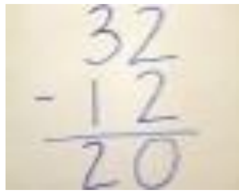
- Add and subtract numbers mentally, including:
  - A three-digit number and ones
  - A three- digit number and tens
  - A three- digit number and hundreds
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction

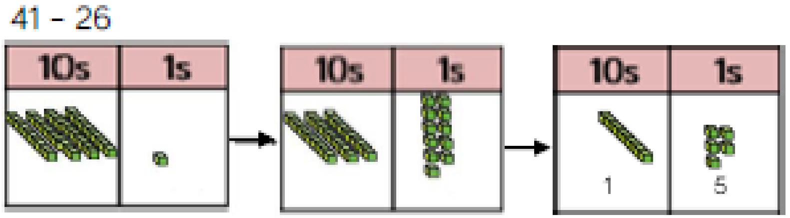
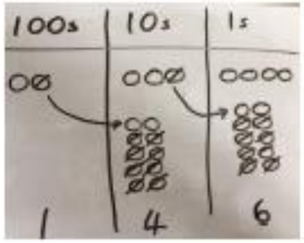
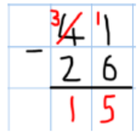
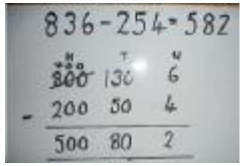
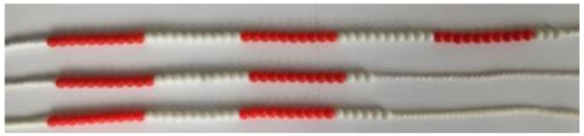
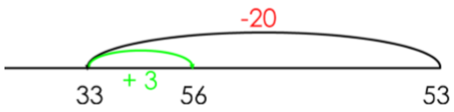
Potential barriers/misconceptions:

- Children sometimes begin subtracting with the left hand column first
- In tens and ones and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. 945-237 = 712
- When the teacher uses the phrase 'near multiple of ten' for mental strategies children often get confused with needing to use multiplication as the operation.

Mental strategies (All calculations must also use missing number problems □□):

- Use number bonds to mentally subtract a one-digit number from:
  - a two-digit number within 100 with or without regrouping (ten as the middle stage: 62-7 = 62-2-5= 60-5 = 55)
  - a three-digit number within 1000 with or without regrouping in tens and ones
  - tens from a three-digit number within 1000 with or without regrouping in hundreds into tens
  - Hundreds from a three-digit number without regrouping
- Subtract a single digit from a multiple of 100. (600-7=593) (600-ð=593)
- Subtract a pair of multiples of 10, crossing 100. (120-30= 90) (ð – 30 =90)
- Subtract a multiple of 10 from a two-digit number crossing 100 (112-30=82) (112-ð=82)
- Subtract a pair of multiples of 100 crossing 1000 (1500-800= 700) (1500-ð=700)
- Subtract 100 from any three-digit number, without crossing 1000 (809-100= 709) (ð-100=709)
- Consolidate subtracting a single digit from a 'teens' number, crossing 10 (use two steps and cross ten as the middle stage: 15-8 = 7 I know this because 15-5-3 = 10-3= 7)
- Find pairs of numbers with a difference of 29, 16.
- Find the difference between two numbers that are close together by counting up. (504-498 = 2+4=6) (1003-992= 992+8+3=1003 = 11)
- Mentally subtract 9,19,29… or 11,21,31 from any two-digit number without crossing 100
- Develop and recognise a pattern such as 68-5=63, 68-15=53, 68-25=43 therefore 68-45=23
- Say the subtraction fact corresponding to a given addition fact: 56+27=83 therefore 83-27=56

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Draw representations to support understanding	Intermediate step may be needed to lead to clear subtraction understanding.  

Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.	Represent the base 10 pictorially, remembering to show the exchange.   234-88 = 146	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.  Begin by partitioning into place value columns Then move to formal method.  
Subtracting near multiples of 10 and compensating	Subtracting tens and adding extra ones Pupils must be taught to round the number that is being subtracted. Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones. 	 53 – 17 = 36	53-17=36

Year 4 – Subtraction (when planning ensure you track forwards to Year 5)

National Curriculum:

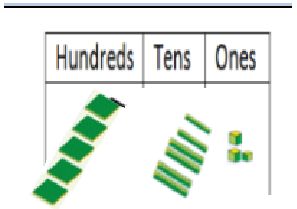
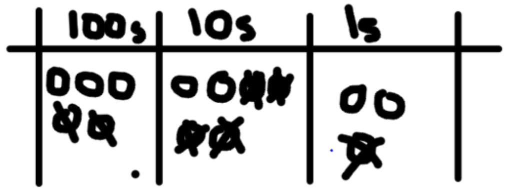
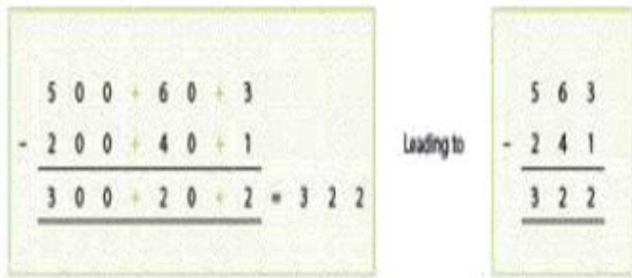
- Add and subtract numbers with up to 4 digits using the formal written methods of column addition and subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two step word problems in context, deciding which operations and methods to use & why

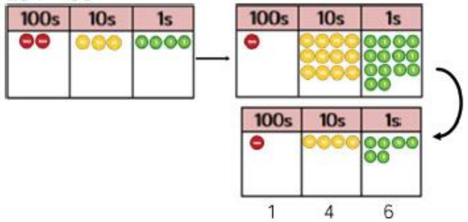
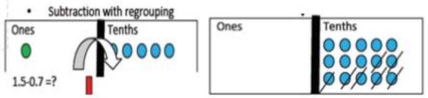
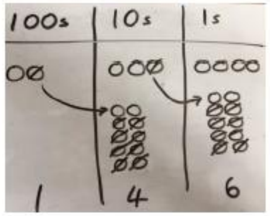
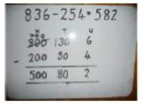
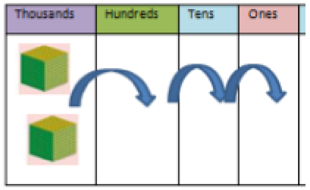
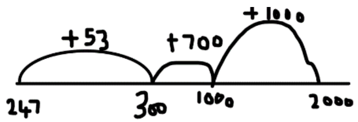
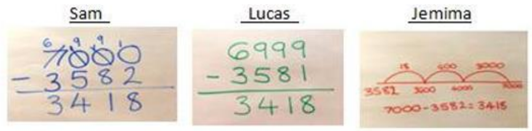
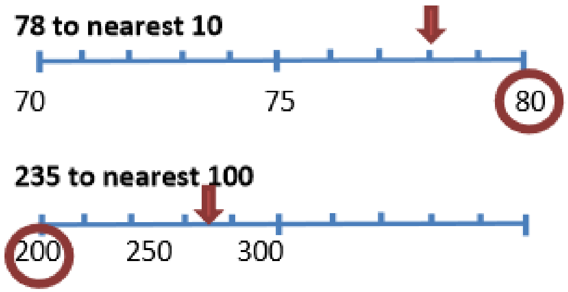
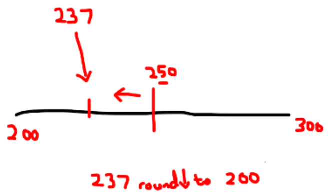
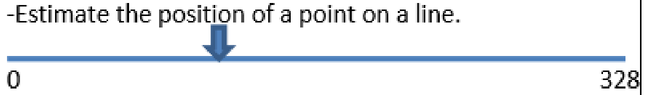
Potential barriers/misconceptions:

- When using the column method pupils sometimes begin subtracting with the left hand column first.
- In tens and ones and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. 945- 237 = 712
- Children may have been incorrectly told 'you can't take a big number away from a small number'. This will cause misconceptions when children start to work in negative numbers. Pupils don't use estimation skills to predict answer
- Lack of understanding around value of decimal numbers
- Forgetting to include or line up decimal point

Mental strategies

- Consolidate knowing by heart all addition and subtraction facts to 20; e.g. all the pairs for 15: 10+5=15, 5+10=15, 9+6=15, 6+9=15, 8+7=15, 7+8=15 and 15-5=10, 15-10=5, 15-6=9, 15-9=6, 15-7=8, 15-8=7 Know how many steps are taken forwards (+) or backwards (-) when moving on a numberline. i.e. To get from 18 back to 6.
- Derive quickly related facts: 160-90=70 therefor 1600-900=700 (1.6-0.9=0.7)
- Find the difference by counting up through the next multiple of 10, 100 or 1000. i.e. count from smaller to larger number i.e. 483-386 Count back in repeated steps of 1, 10, 100, 1000 from any given number. i.e. 2003-8=1995 (counting back in 1s from 2003) or 387-50=337 (counting back in 10s from 387)
- Partition into hundreds tens and ones: 98-43 = 98-40-3= 55
- Subtract the nearest multiple of 10, 100 or 1000 and adjust. i.e. 9, 19, 29 or 11, 21, 31 etc (84-19= 65 because 84-20+1=65) (128-67=61 because it is 128-70+3=58+3=61)
- Use the relationship between addition and subtraction (*If I know 36+19=55 then I also know: 19+36=55, 55-36=19, 5519=36*).
- Work out mentally one fact: (91-25=□) and then state the other three related facts
- Subtract two-digit multiples of 10 (130-50=□)
- Subtract a pair of multiples of 100, crossing 1000 (□-600=900)
- Subtract a multiple of ten from a two or three-digit number without crossing hundreds (76-□=36)
- Subtract a single-digit from a multiple of 10 or 100 (4000-3=□ or □-3=4997)
- Subtract a single-digit from a three or four-digit number crossing tens (7003-6899=□ or 5952-□=5949)
- Find a small difference between a pair of numbers lying either side of a multiple of 1000 (7003-6988=15 by counting up 2 from 6988 to 6990 then 10 to 7000, then 3 to 7003)

Objective & Strategy	Concrete	Pictorial	Abstract
To subtract up to 4 digit numbers (no regrouping) *Introduce decimal subtraction through context of money*	Model process of exchange using Numicon, base ten and then move to place value counters.  	563-241  	563 – 241 = 322 

To subtract up to 4 digit numbers with regrouping	Column method using place value counters. 234 – 88  1.5 – 0.7 = 15 tenths – 7 tenths = 8 tenths = 0.8 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. $\begin{array}{r} {}^{5}\cancel{6}\,{}^{12}\cancel{3}.{}^{1}0 \\ 2\;\;\;\;\;\;\;\;\;\; \\ -\;\;2\;\;6\;.\;5 \\ \hline 2\;\;3\;\;6\;.\;5 \end{array}$ Formal colum method. Children must understand what has happened when they have crossed out digits. $\begin{array}{r} {}^{2}\cancel{2}{}^{1}34 \\ -\;\;88 \\ \hline 6 \end{array}$
To subtract with numbers that have zero	2000-247=  (although shown visually in the column this could also be solved using the number line)	  2000 – 247 = 1753	Sam, Lucas and Jemima are solving the calculation 7000 – 3582 Here are their methods.  Who is correct? Can you explain how each child has reached their answer? Whose method is the most efficient? Use the different methods to solve 4000 – 2831
To round to the nearest 10/100 to estimate answers	 	 	-Estimate the position of a point on a line.  -Use rounding to estimate answer. 328 - 189 = 330 - 200= 130 (actual answer: 139)

Year 5 – Subtraction (When planning ensure you track forwards to Year 6)

National Curriculum:

- Add and subtract whole numbers with more than 4 digits, including formal written methods (column addition and subtraction)
- Add and subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy. Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Potential barriers/misconceptions:

- Misconceptions can occur when decomposing from a 'high' number. e.g. 9000 - 3654. Some pupils will attempt subtraction calculations using the formal written method, failing to recognise that it would be more efficient to calculate the answer mentally.
- Misconceptions occur when pupils (and teachers) use inaccurate language e.g. 2367-
  1265 When talking about 2000 – 1000 they may refer to this as 2-1, unaware of the place value of each number.
- Children can often misplace the decimal point when subtracting decimal numbers.

Mental strategies:

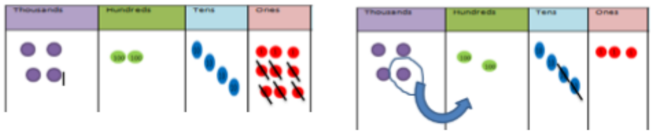
- Derive quickly related facts such as: 150-80=70, 1500-800=700 and 1.5-0.8=0.7
- Find a difference by counting up through the next multiple of 10,100 or 1000 (8006-2993=□ count up from the smaller to the larger number)
- Subtract the nearest multiple of 10, 100 or 1000 and adjust (4005-1997= 2008 because it is 4005-2000+3=2008)
- Recognise that knowing a fact such as 136+319=455 makes it possible to find 455-318 and 455-137
- Work out mentally one fact such as 101-25 and be able to state the three other facts in the number family
- Given the numbers 135, 228 and 363 say or write the four different sentences relating to these numbers
- Subtract multiples of 10 and 100 (620-380=□ and 6200-3800=□)
- Subtract a single digit multiple of 100 from a four digit number crossing 1000 (1263-400=□)
- Subtract a three digit multiple of 10 from a three digit number without crossing the hundreds boundary (742-210=□, 742-□= 532, □-210=532)
- To find what to add to a three digit number to make the next higher multiple of 100. (651+□=700)
- Find what to add to a decimal with units, tenths and hundredths (5.71+ □ = 7) Find the difference between a pair of numbers lying either side of a multiple of 1000 (8004- □=19)
- Subtract a pair of decimal fractions each less than 1 and with up to two decimal places (0.7-0.26)

Objective & Strategy

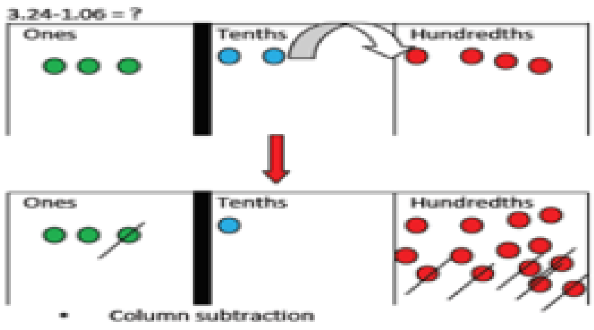
Subtract with at least 4 digits, including money and measures (regrouping in the 1000s, 100s, 10s and 1s). Identify common misconceptions.

Concrete

Track back to year 4 for progression

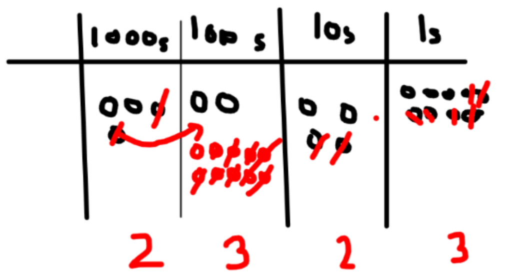


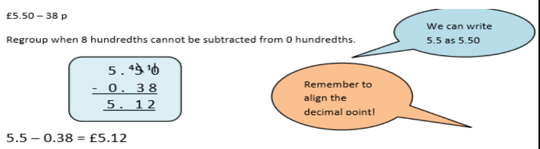
When we subtract 1926 from 4249 we get 2323



Pictorial

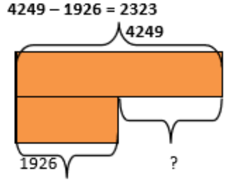
Children to draw place value counters and show their exchange



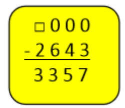


Abstract

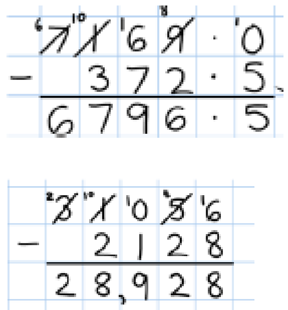
4249 – 1926 = 2323



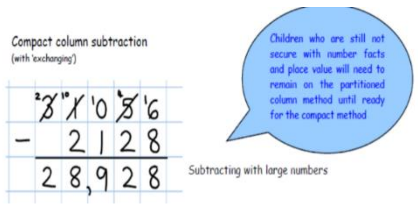
Use the inverse to find the value in the box:



3357 +2643 = 6000 (the missing value is 6)



Use zeros for place holders.



Year 6 – Subtraction (When planning ensure you consolidate all methods from KS2)

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National Curriculum:

- Perform mental calculations, including with mixed operations and large numbers.
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

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Potential barriers/misconceptions:

- Pupils without a strong foundation in place value will continue to make mistakes with column subtraction. These are not 'careless mistakes' but fundamental misconceptions.
- When subtracting with decimals such weaknesses are highlighted because of the decimal point.
- Pupils are uncertain about the order of operations when carrying out calculations.
- Pupils are unable to accurately estimate and use the inverse to check.

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Mental strategies (building on mental strategies from Y5):

- To find the difference by counting up through the next multiple (count up from the smaller to larger number mentally: 8000-2785 is 5 +10+ 200+ 5000 = 5215
- Subtract 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 by subtracting 1,2,3 then adjusting by 0.1
- Work out mentally one fact 4.97-1.58 and then state three other related facts Subtract four digit+ multiples of 100 (570,000 + 250,000= □)
- Find what to add to a decimal with units, 10ths and 100ths to make the next higher whole number or 10th.
- Subtract a pair of decimal fractions each less than 1 and with up to two decimal places.
- Subtract numbers with different numbers of digits.
- Find the difference between 4387 and 782 = 175

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<tr>
<th>Objective & Strategy</th>
<th>Concrete</th>
<th>Pictorial</th>
<th>Abstract</th>
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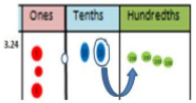
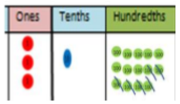
<tr>
<td>Subtract with increasingly large and more complex numbers (refer back to year 5) and decimal values.</td>
<td>

3.24 – 1.06= 2.18



3 ones 2 tenths 4 hundredths
3 ones 1 tenth 14 hundredths

First subtract the hundredths



3 . ¹2 ¹4
- 1. 0 6
8

14 hundredths – 6 hundredths = 8 hundredths
Then subtract the tenths:

3 . ¹2 ¹4
- 1. 0 6
. 1 8 (1 tenth – 0 tenths = 1 tenth)

Lastly subtract the ones:

3 . ¹2 ¹4
- 1 . 0 6
2 . 1 8

3 ones – 1 one = 2 ones

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Use own place value grids and drawings to reinforce conceptual understanding

4,249 - 1,926 = 2323

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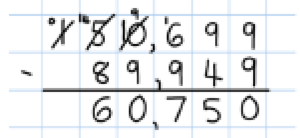

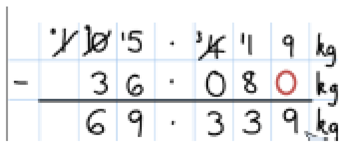

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

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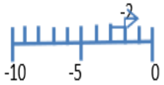
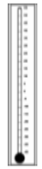
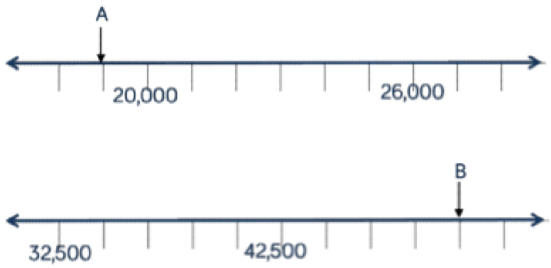
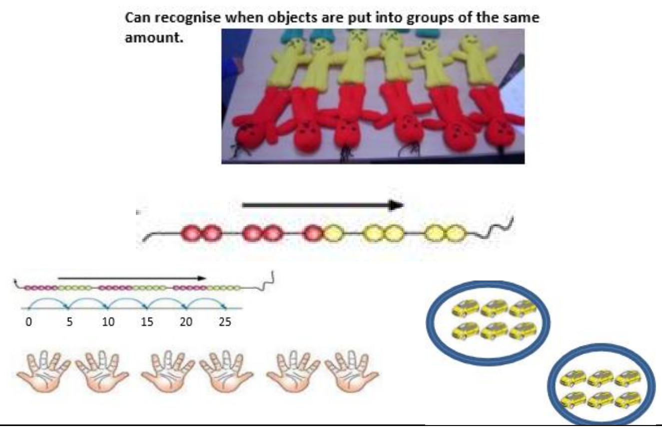
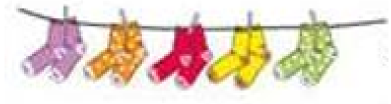
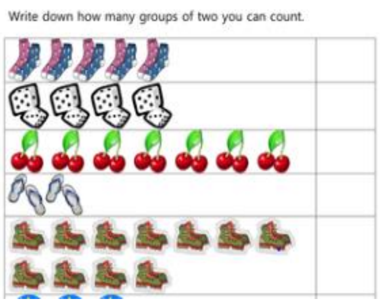
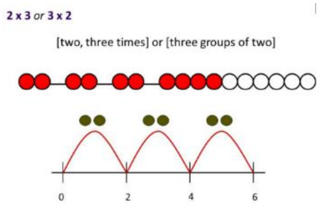
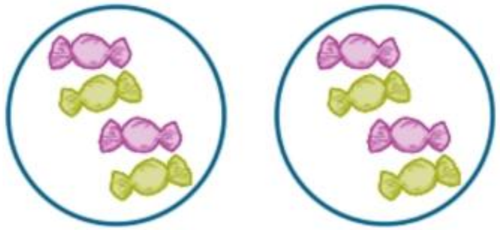
To understand the order of brackets and use estimating to check answers	There were 94 players in Arsenal juniors. Last year 21 players left and 39 joined. How many players are there now in Arsenal Juniors? $\underbrace{94-21}_{73}+39$ $73+39$ = **112** players are now in Arsenal juniors.  Standard order of operations: 1) Do the operation in brackets 2) Do operation involving indices 3) Multiply and divide in order from left to right. **4) Add and subtract in order from left to right.** How would you use a bar model to represent this word problem?	(-3) - (-2) = -1   **Subtracting** a negative number is the same as adding: eg (−5) − (−2) is the same as (−5) + 2 = −3 	9 - 4 + 3 = Calculating from left to right: 9 - 4 + 3 = 5 + 3 = 8
To find the difference	Find the difference between A and B. 	A=19,000 B=50,500 50,500 − 19,000 31,500 or 	50,500 – 19,000 = 31,500 The difference between A and B is 31,500 At this stage children should be choosing the most efficient methods.

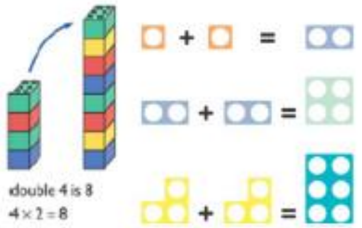
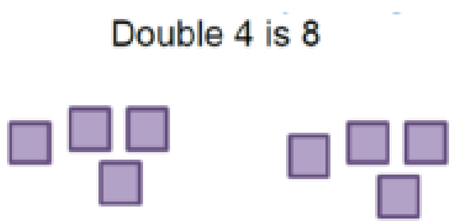
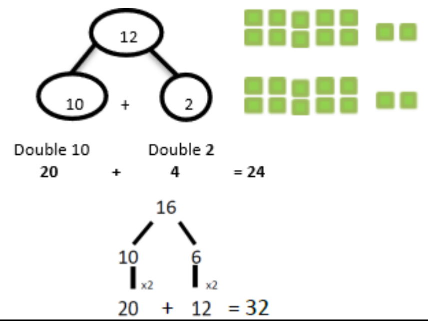
Multiplication

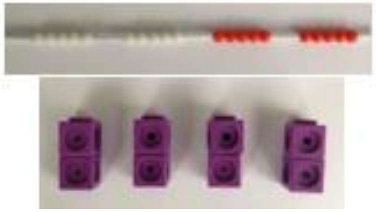
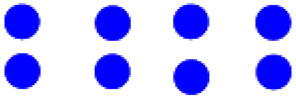
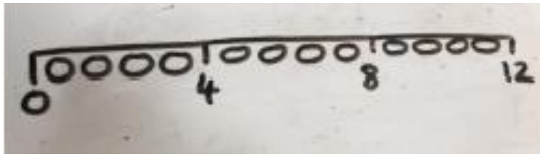
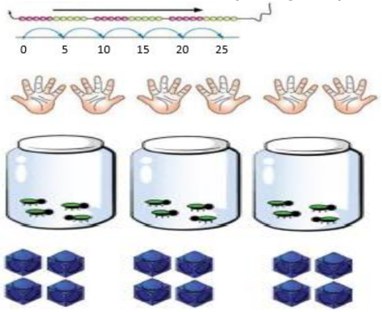
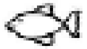
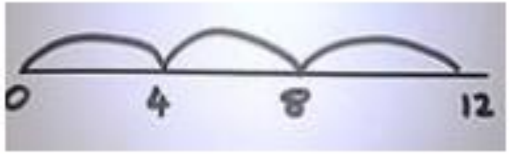
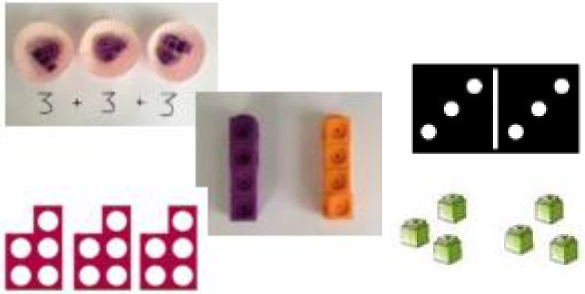
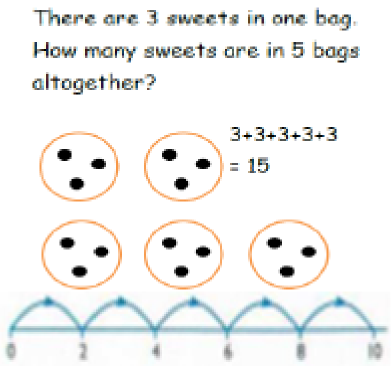
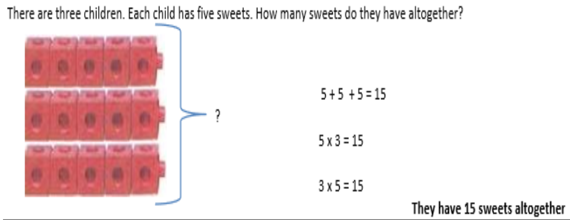
EYFS – Multiplication (when planning ensure you track forwards to Year 1)

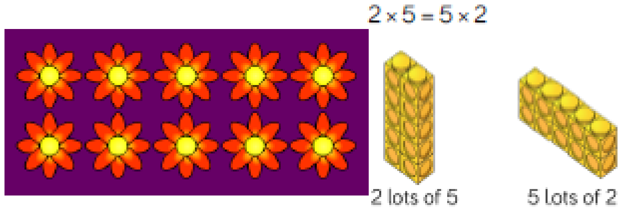
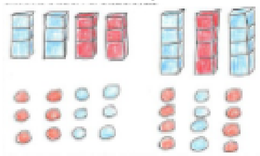
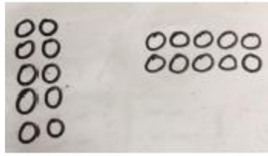
Early Learning Goal 11:	Potential barriers/misconceptions:	Mental Maths (can revisited throughout day once concept has explicitly shared):
• Using quantities and objects, they add two single-digit numbers and count on or back to find the answer. • They solve problems, including sharing, doubling and halving.	• Children inaccurate when displaying arrays of cubes/objects and so pattern is not clear. Link not clear between the array and the seemingly abstract number given as the answer. Children unable to place objects in equal groups. • Not secure with one to one correspondence counting in ones, therefore will be unable to count pairs accurately • When counting orally in 10s: 60,70,80 follow a regular pattern which link to single digit numbers however 10, 20, 30 do not. • Conceptual understanding of 'same' and 'different' is not secure (both language and concept).	• Count in tens (recite the sequence ten, twenty, thirty… one hundred.) • Do the same backwards. • Count on and back in tens from a given tens number Say the tens number that goes before or after a given tens number (*when you count in tens, what number comes before 60? 90?)* • Count from a given tens number and stop at another (count on in tens from 20 and stop at 70, count back in tens from 60 and stop at 30) • Count around in a circle of children, starting with Abdul on 20, who do you think will say 70? • Understand odd and even numbers linked to getting 'into pairs'. Count pairs: children, socks, animals in the ark, eggs in an egg box

Objective & Strategy	Concrete	Pictorial	Abstract
Can recognise equal groups	**Use number beads to count in equal groups of 2,5 and 10** 		Playdough men 6 + 6 =12 2 groups of 6 equals 12 6 groups of 2 equals 12 6+6=12 2+2+2+2+2+2=12 2x6=12 6x2=12
			3x2=6 2x3=6 *Six is even because it can be shared equally into groups of 2. 3 is odd because is cannot be shared equally.*

Year 1 - Multiplication (when planning ensure you track back to ELFS to ensure progression)		
National Curriculum: • Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	**Potential barriers/misconceptions:** • Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary for example 'multiplied by' • When objects placed in arrays it may be done inaccurately therefor link between arrays and answers unclear • Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones. • Don't understand how 'turning the grid around' shows that multiplication can be done in any order	**Mental Maths:** • To count in twos, fives and tens • Count forwards and backwards in 2s from any given number • Count forwards and backwards in 5s from any given number • Count forwards and backwards in 10s from any given number • Recognition of all odd and even numbers • Rapid recall of doubles to 10 (and corresponding halves) • Rapid recall of doubles to 20

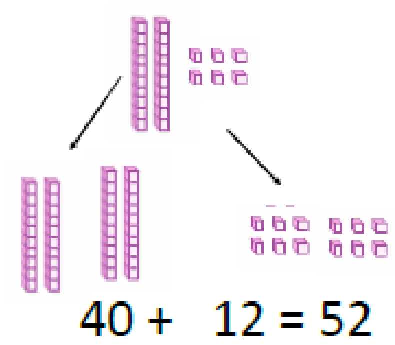
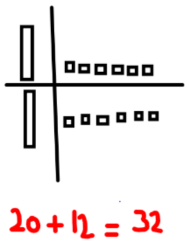
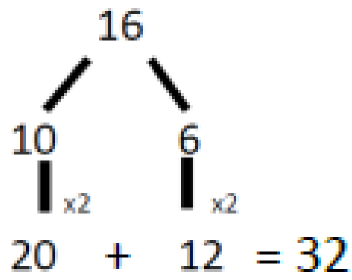
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 

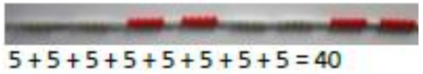
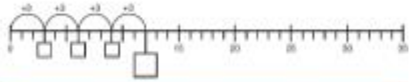
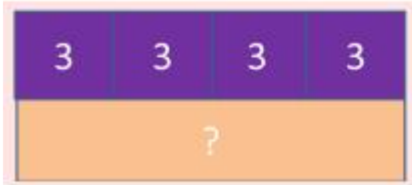
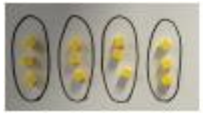
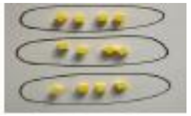
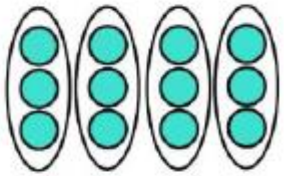
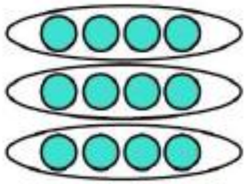
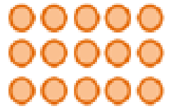
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	Use manipulatives to create equal groups. 	Draw and make representations Draw  to show 2 x 3 = 6	$3 \times 4 = 12$ 
Repeated addition	Use different objects to add equal groups 	Use pictorial including number lines to solve problems 	Write addition sentences to describe objects and pictures.  

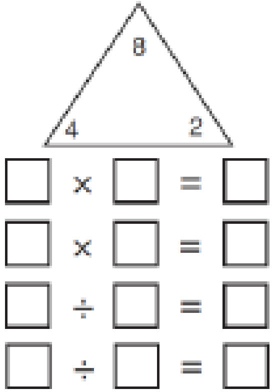
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.  	Draw representations of arrays to show understanding  	3 x 2 = 6 2 x 5 = 10

Year 2 - Multiplication (track back to year 1 to show progression)

National Curriculum:	**Potential barriers/misconceptions:**	**Mental Maths:**
• Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs • Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	• Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones. • Don't understand how 'turning the grid around' shows that multiplication can be done in any order. • Not understanding that multiplication is repeated addition.	• Rapid recall of 2,5 and 10 times tables • Count in 5s clockwise around a clock face/ anticlockwise around a clock face • Count forwards and backwards in 2s, 5s and 10s from any given number. Recognition of all odd and even numbers • To recall related multiplication and division facts linked to other multiplication tables (3x4=12, 4X3=12, 12÷4=3, 12÷3=4) • Rapid recall of doubles and their corresponding halves (double 12 is 24, half 24 is 12) • Rapid recall of half of all two-digit even numbers (half of 12, 18, 42 etc.) Recognise that multiples of 10 end in 0, 5 end in 5 and 0, 2 end in 0,2,4,6,8. Recognise two-digit multiples of 10,5,2 (65 is a multiple of 5, 72 is a multiple of 2, 50 is a multiple of 5 and 10) • Work out the four times table by doubling the two times table. • Multiply a single digit by 1 or 10 (3 x 1 = 3, 7x10=70 etc.) • Multiply a single digit up to 5 by 2,3,4,5. (2x3=□ 4x4=□)

Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling	Model doubling using dienes and PV counters. 	Draw pictures and representations to show how to double numbers Double 16 	Partition a number and then double each part before recombining it back together. 

Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.     	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$
Multiplication is commutative	Create arrays using counters and cubes and numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$

Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$	Show all 8 related fact family sentences.

Year 3 - Multiplication (track back to year 2 for progression)

National Curriculum:

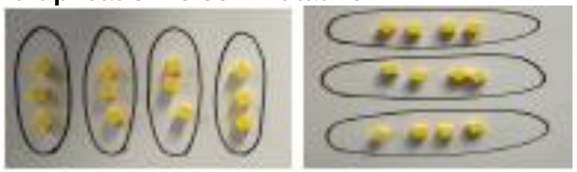
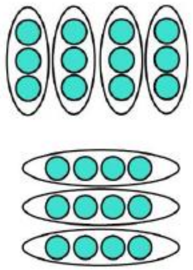
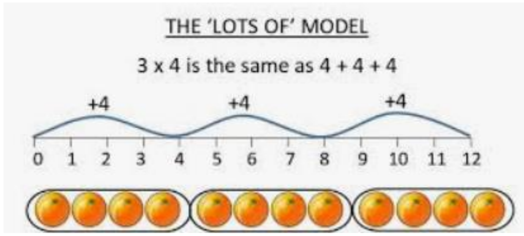
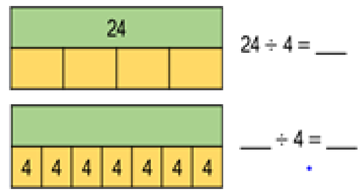
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

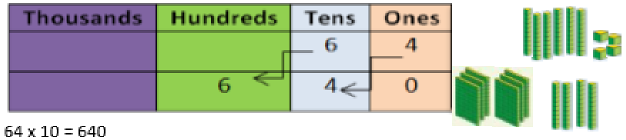
Potential barriers/misconceptions:

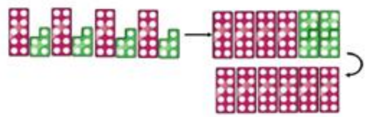
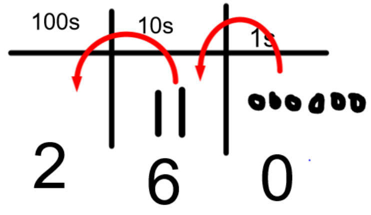
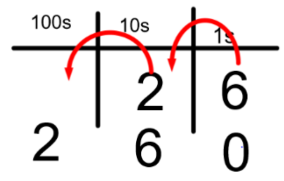
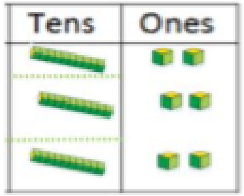
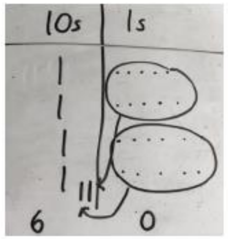
- Children may need to go back to multiplication as an array, or repeated addition to gain security
- Some children struggle to apply partitioning and recombining when multiplying. e.g. 14 x3 is calculated as *(10x3) + 4 = 34 Or 14 x 3 = 312,* when they should do *(10x3) + (3x4) = 30+12=42*
- Lack of confidence with place value sees confusion in the value of the two digits
- Children are incorrectly taught that x10 involves 'adding a zero' rather than developing understanding of place value
- Also unable to see that x100 is the same as x10 and x10 again.

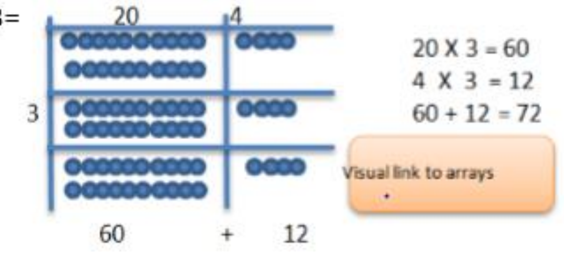
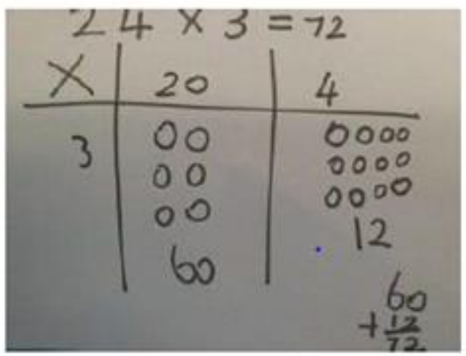
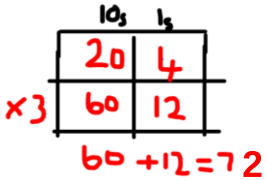
Mental Maths:

- Rapid recall of 3, 4 and 8 times tables
- Count forwards and backwards in 3s from any given number
- Count forwards and backwards in 4s from any given number
- Count forwards and backwards in 8s from any given number
- To use the 2,5 and 10 times table to derive other multiplication facts (*if I know 2x5= 10 I also know 20x5=100*)
- To know doubles of all numbers up to 50
- To know doubles of all multiples of 5 up to 100
- Observe the effect of multiplying by 10
- Multiply any single digit by 1, 10, 100 and 0
- Multiply a two-digit number by 2,3,4, or 5 without crossing the tens boundary. (11x5, 23x2)
- Check halving with doubling
- To multiply multiples of 10 with one-digit number

Objective & Strategy	Concrete	Pictorial	Abstract
Equal groups Repeated addition Using the inverse	**Tract back to year 2 – arrays, to show equal groups and multiplication is commutative**  Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	  Circle the buttons in groups of 4.   Can you also split the buttons into 4 equal groups? How is this the same? How is it different? **There are _ equal groups of _** **There are _ equal groups of 4**	3x4=12 4x3=12 12÷3=4 12÷4+3 **use the inverse** 

<table>
<tr>
<td>To understand how place value changes when multiplying by 10</td>
<td>
64 x 10 = 640
When multiplying by ten the original number moves one place to the left.
It gets ten times bigger(X10).
Zero acts as a place holder in the ones.

Partition to multiply using Numicon, base 10 or Cuisenaire rods.
4 x 15
</td>
<td>26 x10
</td>
<td>26 x10
</td>
</tr>
<tr>
<td>Partition into 10s and 1s</td>
<td>
Then multiply the tens by 3
1 2
X 3
3 6
1 ten x 3 = 3 tens</td>
<td>Children to represent the concrete manipulatives pictorially.
</td>
<td>4x15=60

4 x 10 = 40
4 x 5 = 20
40 + 20 = 60</td>
</tr>
<tr>
<td>Introduce grid method for two-digit by a one-digit number</td>
<td>24 x 3=
</td>
<td></td>
<td>24 x 3=72

This may lead onto the formal method which is taught alongside pictorial methods, but children are not expected to be secure in this method until year 4.

2 4
x 3
7 2
1</td>
</tr>
</table>

Year 4 – Multiplication (track back to year 3 for progression)

National Curriculum:

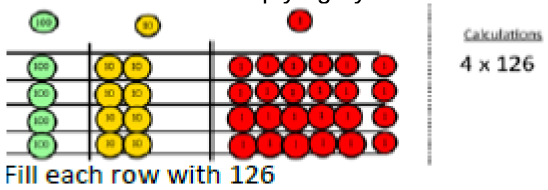
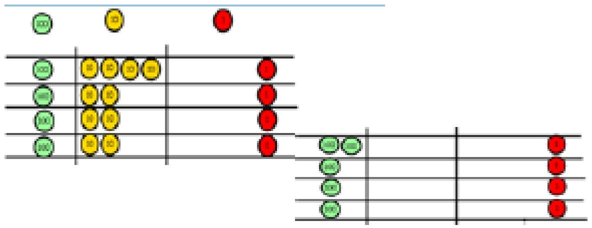
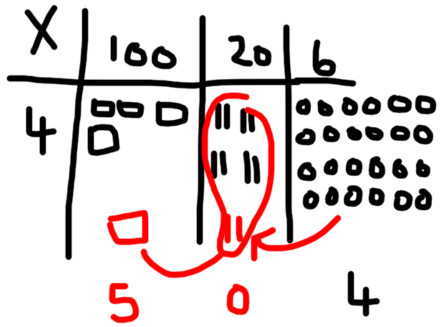
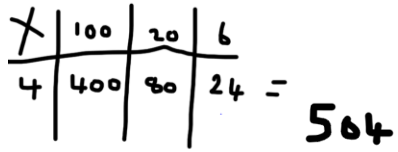
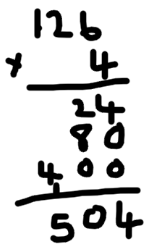
- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as *n objects are connected to m objects*

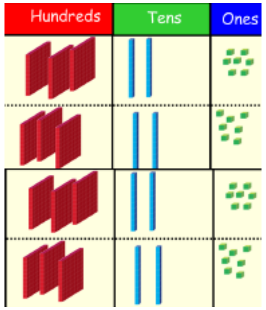
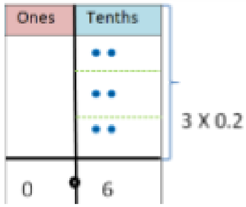
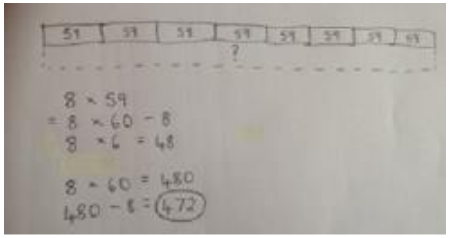
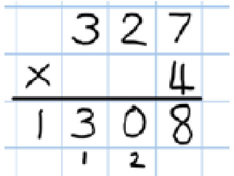
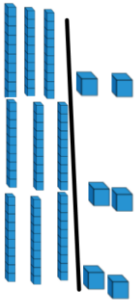
Potential barriers/misconceptions:

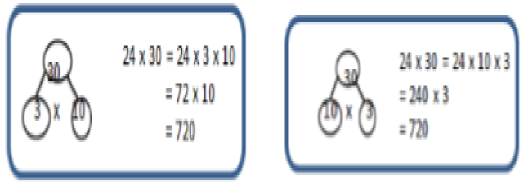
- Children unclear around language of multiplication: multiple, factor
- Children not secure in their rapid recall of times tables facts and conceptual understanding
- Some children not using known facts and commutative properties to solve calculations such as 8x□=48
- Children may not see the link between known facts and multiplying multiples by 10 and 100
- Children are incorrectly taught that x10 involves 'adding a zero' rather than developing understanding of place value
- Also unable to see that x100 is the same as x10 and x10 again

Mental Maths:

- Rapid recall of all numbers multiplied by 10, 100, 1000
- Rapid recall of all multiplication and division facts up to 12 x 12
- To understand what happens when multiplying by 1 and 0
- To multiply together three numbers
- To know by heart all doubles and halves (double 34 is double 30 + double 4 = 60+8= 68)
- To multiply by 4 (double and double again: 7x4 = double 7= 14. Double 14 = 28)
- To multiply by 5 (multiply by 10 and halve: 5x9 = 10x9= 90 halved = 45)
- To multiply by 20 (multiply by 10 and double) Work out 8 times table by doubling four times table
- Use doubling to work out multiples of 15. (1x15= 15, 2x15= 30, 4x15=60, 8x15=120, 16x15=240)
- Use combinations of these facts to find e.g. 11x15 (8x15 + 2x15 + 1x15 = 120 +30+15 = 165)
- Work out the six times table by adding 2 times table facts and 4 times table facts. To multiply a number by 9 or 11, multiply it by 10 and add/subtract the number (14x9 = 140-14 = 126 and 14x11= 140+14= 154)
- To know the three corresponding number facts when given a multiplication number sentence.

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for two-digit x one-digit Move to multiplying three-digit numbers by one-digit	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows   Add up each column, starting with the ones making any exchanges needed		126x4=504   link to expanded method

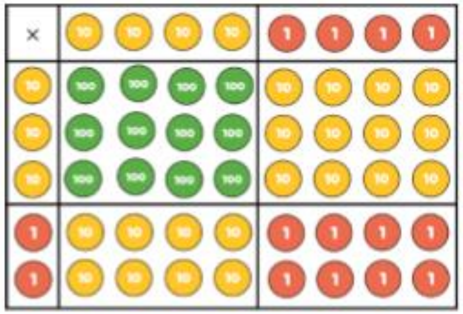
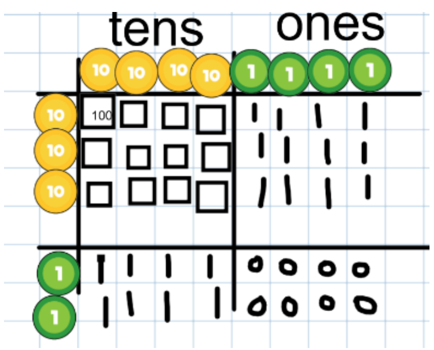
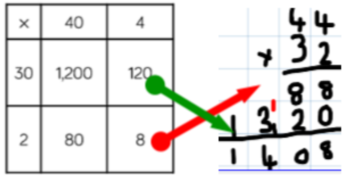
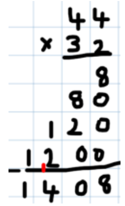
<table>
<tr>
<td>Column multiplication
2 and 3 digits by 1 digit</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642

It is important at this stage that they always multiply the ones first.
0.2 x 3 =
0.2 + 0.2+ 0.2 = 0.6
3 x 2 tenths = 6 tenths = 0.6
0.2
X 3
0.6

The corresponding long multiplication is modelled alongside</td>
<td><table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
The grid method maybe used to show how this relates to a formal written method.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</td>
<td>327
x 4
28
80
1200
1308

This may lead to a compact method.</td>
</tr>
<tr>
<td>To use the distributive law</td>
<td>To use the distributive law: 32x3 = (30x3) + (2x3) = 90+6 = 96

90 6</td>
<td>32 x 3

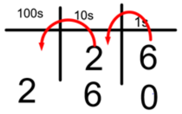
30 x 3= 90 2 x 3 =6</td>
<td>32x 3 = (30 x 3) + (2x3) = 90+6 =96
Find the product of 24 and 30
</td>
</tr>
</table>

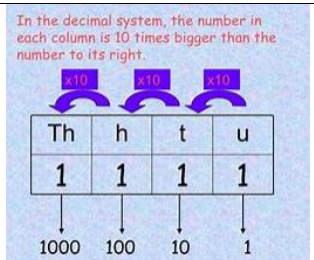
<table>
<tr><th colspan="3">Year 5/6 – Multiplication (track back to year 4 for progression)</th></tr>
<tr>
<td><u>National Curriculum:</u>
• Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
• Know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
• Establish whether a number up to 100 is prime and recall prime numbers up to 19
• Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two digit numbers
• Multiply numbers mentally drawing upon known facts
• Multiply whole numbers and those involving decimals by 10, 100 and 1000
• Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
• Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
• Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
• Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates</td>
<td><u>Year 5 Potential barriers/misconceptions:</u>

• Children may struggle to partition a two-digit number into tens and ones correctly for whole numbers and tenths for decimals.
• Children believe that multiplication always increases a number. This is only when a positive number is multiplied by a whole number greater than 1.</td>
<td><u>Mental Maths:</u>
• To find all factor pairs of a number & find common factors of two numbers
• To establish whether a number up to 100 is prime
• To recall prime numbers up to 19 Recognise 1,4,9,16,25,36,49,64,81,100 as square numbers (relate to drawings of squares)
• Find all the pairs of factors for any number to 100 (pairs of factors to 36 are 1&36, 2&18, 3&12, 4&9, 6&6)
• Use factors for finding products mentally (16x12 = 16 x 3x2x2= 48 x2x2 = 96 x2 = 192)
• To double using known facts (double 79 = double 70 + double 9 = 140+18= 158)
• Double a number ending in 5 and halve the other number (16x5 is equivalent to 8x10=80)
• To multiply by 50 (multiply by 100, then halve: 26 x 50 = 26 x 100 = 2600 halved = 1300)
• Calculate 16 times table by doubling 8 times table facts
• Calculate 25 times table by doubling: (1x25=25, 2x25=50, 4x25=100, 8x25=200, 16x25=400 use combinations of these facts to work out e.g. 25x25 = (16x25) + (8x25) + (1x25) = 625
• Work out 12 times table by adding 2 times table and 10 times table
• To multiply a number by 19 or 21, multiply it by 20 and add or subtract the number (13x21 = 13x20+13= 273)</td>
</tr>
<tr>
<td><u>Year 6 National Curriculum:</u>
• Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
• Perform mental calculations, including with mixed operations and large numbers
• Identify common factors, common multiples and prime numbers
• Use their knowledge of the order of operations to carry out calculations involving the four operations
• Solve problems involving addition, subtraction, multiplication and division
• Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.</td>
<td><u>Year 6 Potential barriers/misconceptions:</u>
• Misconceptions can arise when multiplying decimals
• Belief that if 1x1=1 then 0.1x0.1=0.1 (this is ⅒ x⅒ which is one tenth ‘of’ one tenth which =1/100= 0.01)
• Interpreting a multiplication number sentence; 2x6 is often interpreted as the same as 6x2. 2x6 is ‘2 multiplied by 6’, 2 taken six times or 2+2+2+2+2+2. The first number is ‘operated on’.
• 6x2 would be ‘6 multiplied by 2’, 6 taken twice or 6+6. ‘Everyday’ interpretation (referred to throughout programme) can be different as 2x6 is referred to as 2 ‘times’ 6 or 2 ‘groups of’ 6.
• This is not a problem as it is commutative and both will give the answer 12.</td>
<td><u>Mental Maths:</u>
Rapid recall of all multiplication tables up to 12 x 12 (and derive corresponding division facts)
Identify common factors, multiples and prime numbers
Know the squares of all numbers from 1x1 to 12x12
Derive quickly squares of multiples of 10 to 100, such as 20^2, 80^2.
To double decimal numbers
Double all multiples of 10 up to 1000
Double all multiples of 100 up to 10,000
Use related facts to double (double 277 = 400 + 140 + 14 = 554)
Double a number ending in 5 and halve the other number (14x5 = 7x10= 70)
Halve/double one number in the calculation, find the product then double/halve it
To multiply by 15 (multiply by 10, halve the result then add the two parts together: 22x15 = 22x10=220+110=330)
To multiply by 25 (multiply by 100 and then divide by 4)
To know the 24 times table (six times table, double and double again – or double 12x)
To calculate 17 times table (add seven times table and ten times table)
To multiply a number by 49 or 51 (multiply it by 50 and add or subtract the number)
To multiply a number by 99 or 101 (multiply by 100 and add or subtract the number)</td>
</tr>
</table>

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication for three and four-digits x one-digit.
Track back to Year 4

Multiply two-digits, three-digits and four-digits by two-digits.</td>
<td>44 x 32
</td>
<td>
</td>
<td>44x32

Or long multiplication
4 x2
40 x 2
4x30
40x30
</td>
</tr>
<tr>
<td>To multiply whole numbers& decimals by 10, 100, 1000

Rounding to check the answer</td>
<td>5.928 x 10 = 59.28
X10
<table>
<tr><th>Tens</th><th>Ones</th><th>Tenths</th><th>Hundredths</th><th>Thousandths</th></tr>
<tr><td></td><td>5</td><td>9</td><td>2</td><td>8</td></tr>
<tr><td>5</td><td>9</td><td>2</td><td>8</td><td></td></tr>
</table></td>
<td>26 x10



</td>
<td>Multiplying decimals by 100 and 1000:
$0.3 \times 100 = \frac{3}{10} \times 100$
$= 3 \times 10 = 30$
$0.3 \times 1000 = \frac{3}{10} \times 1000$
$= 3 \times 100 = 300$

By rounding the actual values to more manageable numbers, you can estimate the answers to many problems:

£2.99 + £3.10 + 99p ≈ £3 + £3 + £1 = £7

29 × 9 ≈ 30 × 10 = 300

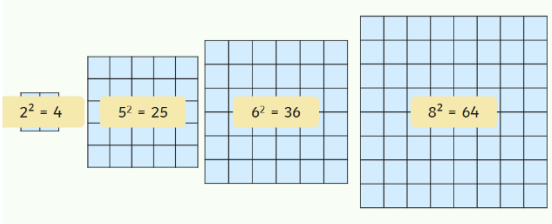
61 ÷ 6 ≈ 60 ÷ 6 = 10
</td>
</tr>
</table>

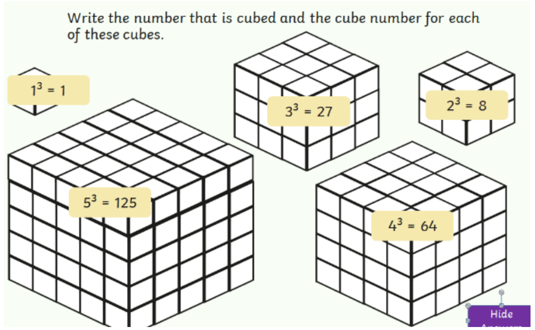
To understand factor, multiple, prime, square and cube numbers and construct equivalence statements

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

How many squares can you draw on a hundred square?

What do you notice?





3^2 means '3 squared', or 3 x 3.
It tells us how many times we should multiply 3 by itself.

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$ etc

The opposite of a square number is a square root.
We use the symbol $\sqrt{\ }$ to mean square root.
So we can say that $\sqrt{4} = 2$ and $\sqrt{25} = 5$.

1, 4, 9, 16, 25… are known as square numbers.

2 x 2 x 2 means '2 cubed', and is written as 2^3

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$…etc.

The opposite of a cube number is a cube root.
We use the symbol $\sqrt[3]{\ }$ to mean cube root.
So $\sqrt[3]{8}$ is 2 and $\sqrt[3]{27}$ is 3.
Each number only has one cube root.

1, 8, 27, 64, 125… are known as cube numbers.

4 x 35 = 2 x 2 x 35

3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10

To multiply fractions

$$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9}$$

$$= \frac{24}{36}$$

$$\text{product} = \frac{2}{3}$$

Multiply the numerators then multiply the denominators.

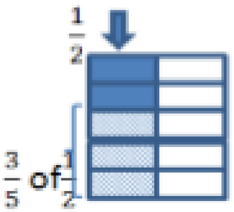
Finally simplify by looking for common factors – both 24 and 36 have a factor of 12.

Find the product

$$\frac{3}{5} \text{ of } \frac{1}{2} = \frac{3}{5} \times \frac{1}{2}$$

$$= \frac{3 \times 1}{5 \times 2}$$

$$= \frac{3}{10}$$



$$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9}$$

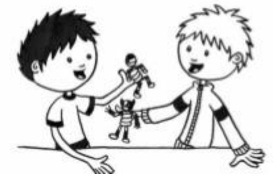
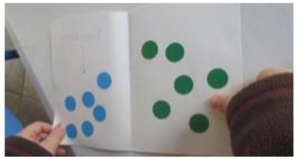
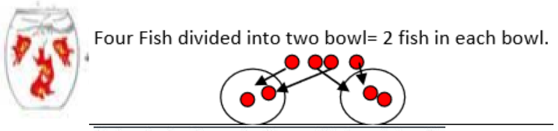
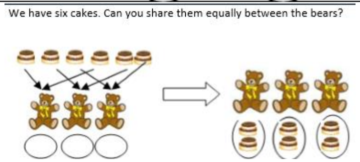
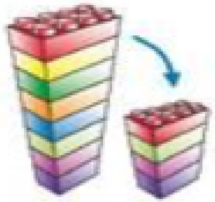
$$= \frac{24}{36}$$

$$\text{product} = \frac{2}{3}$$

Division

EYFS – Division (When planning ensure you track forwards to year 1)

Early Learning Goal 11	**Potential barriers/ misconceptions**	**Mental Maths (can revisited throughout day once concept has explicitly shared):**
• Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. • They solve problems, including doubling, halving and sharing.	• Pupils do not have 1:1 correspondence and incorrectly count the small group of objects • Pupils do not realise that the last number counted represents the total number of objects in the group • When objects are shared between two and four, pupils do not share equally and understand that each group should have the same number of objects • Conceptual understanding of 'same' and 'different' is not secure (both language and concept)	• To count forwards and backwards in 1s • To count forwards and backwards in 2s To count forwards and backwards in 10s • To count forwards and backwards in 1s from any given number • To count in pairs (children, shoes, animals) • Put half of the: sheep in the field, cars in the garage, dinosaurs in the box • To quickly derive: Doubles of numbers 1-10 Halves of even numbers to 20

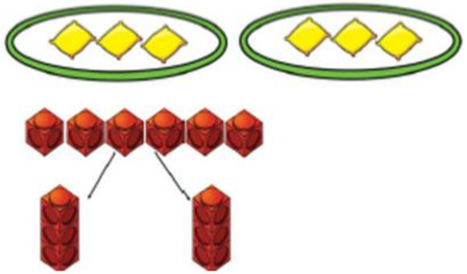
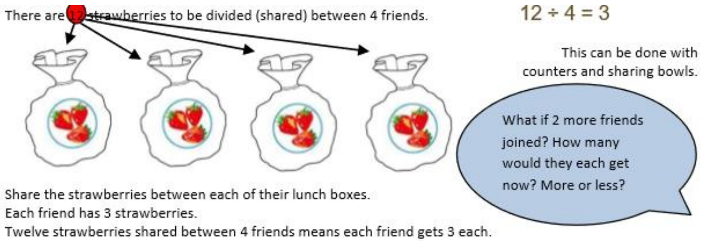
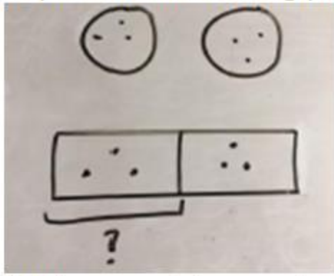
Objective & Strategy	Concrete	Pictorial	Abstract
To share objects equally (with self + others)	  How should we put the seeds in these four pots? Is there a way so that we'll have the same? Are there any left over?  Can we share out these sweets fairly? How shall we do it? Between 2 people? What would happen if it was between 3 people?	 	6 shared by 2 is 3 equal groups
To halve an equal group up to 12	 Half of 8 is 4	 Half of 6 is 3	Half of 6 equals 3

Year 1 Division – (Track back to EYFS to ensure progression)

National Curriculum:
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Potential barriers/ misconceptions:
- Pupils confuse numbers when counting in twos; have difficulty understanding a pair consists of two objects.
- Pupils halve by sharing or forming pairs and counting but may not associate it with division by two or division between two.
- Pupils do not recognise that the division symbol means divide.
- Pupils are not given the opportunity to explore division through both sharing and grouping.
- When dividing by two or four, pupils do not share objects equally as they do not accurately count numbers in each group.

Mental Maths:
- To count forwards and backwards in 2s
- To count forwards and backwards in 5s
- To count forwards and backwards in 10s
- To count forwards and backwards in 2s, 5s and 10s from any given number
- To have rapid recall of numbers up to 20 divided by 2.
- To have rapid recall of numbers up to 100 divided by 10.
- To derive the corresponding division facts when given multiplication fact (number families)
- To quickly derive:
  - doubles of numbers 1-15
  - doubles of 5,10, 15 to 50
  - halves of even numbers to 20
  - halves of even multiples of 10 (20, 40, 60, 80, 100)
  - halves of multiples of 10 up to 100
- To divide a two digit multiple of ten by 1 or 10 (20÷1 = 20 and 50÷10=5)

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	Sharing using a range of objects. 6 ÷ 2   	Represent the sharing pictorially.  Children use pictures or shapes to share quantities. 	6÷2=3 There are 2 groups of 3 2 groups of 3 equals 6 12÷4=3

Year 2 Division – (Track back to Year 1 and forwards to Year 2 for progression)

National Curriculum:

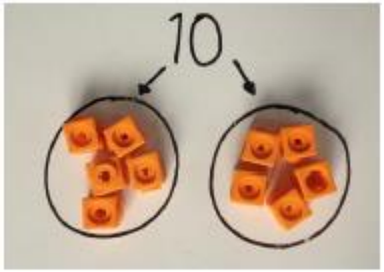
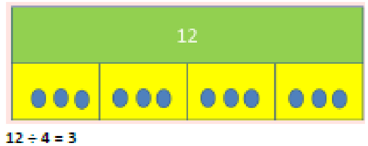
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

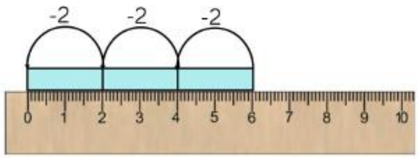
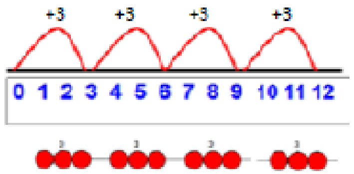
Potential barriers/ misconceptions:

- Pupils confuse the words ‘halving’ and ‘doubling’.
- Pupils do not use knowledge of doubles to find half of a number; for example: continue to find half by sharing using a ‘one for you’ approach and cannot apply knowledge of doubles
- Pupils do not understand that ‘sets of’ or ‘groups of’ need to be subtracted to solve the problem.
- Interpret 12 ÷ 3 as 12 shared between 3 and use objects or pictures to share out the 12 but lose track of their recording as numbers increase due to having no other strategy available such as counting in steps or groups.
- Pupils may not be proficient in counting forwards and backwards in equal steps so make mistakes when carrying out repeated subtraction.

Mental Maths:

- To count in 2s, 5s and 10s forwards and backwards from any given number.
- To have rapid recall of 2,5 and 10 times tables.
- To connect the ten times table to place value.
- To count around the clock face using the five times table.
- To know multiplication facts and corresponding division facts. (2x3=6 therefor 3x2= 6 and 6÷2=3 and 6÷3=2)
- To halve two digit numbers
- To identify half-past the hour using an analogue clock. (knowing that 30 is half of 60)
- Respond rapidly to oral questions phrased in a variety of ways *(share 18 between 2, divide 6 by 3, how many tens make 80? How many £2 coins do you get for £20? How many 2cm lengths can you cut from 10cm ribbon?)*
- Use known facts to derive quickly: doubles of numbers 1-20 doubles of 5,10, 15 to 100 halves of even numbers to 20 halves of multiples of 10 up to 200
- To know that to find a quarter you must halve and halve again (one quarter of 20 is 5, half of 20 is 10 and half of 10 is 5)
- To divide a two-digit multiple of ten by 1, 10 or zero (divide 30 by 1, divide 50 by 10, divide 70 by zero)
- To halve any multiple of ten to 100

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  	12 ÷ 3 = 4

Division as grouping and repeated subtraction	 3 groups of 2	Use number lines for grouping 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Year 3 - Division (track back to year 2 and forwards to year 4 for progression)

National Curriculum:

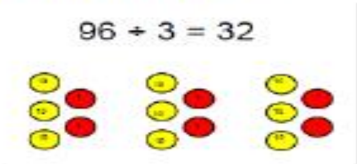
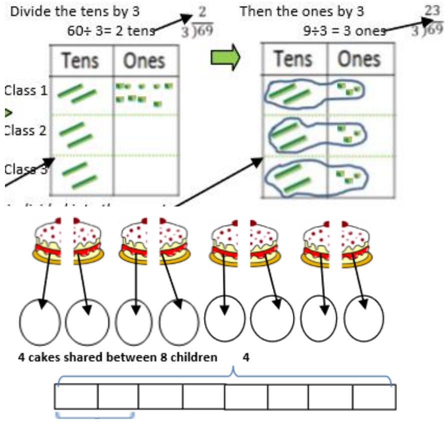
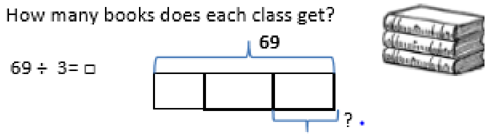
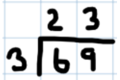
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Potential barriers/ misconceptions:

- Pupils are not secure in multiplication facts for 2,4,5,6,8,10 tables
- Pupils do not recognise the inverse of multiplication is division
- Pupils confuse the order of division and do not recognise that you need to begin with the largest number when writing a number sentence
- Pupils carry out division by sharing or grouping but cannot cope with a remainder and do not recognise that a remainder must always be less than the divisor
- Pupils associate X with multiplication and ÷ with division and can do calculations 8 X 2 and 16 ÷ 2 but are not able to find missing numbers in statements such as 6 X□= 12 and □- 5 = 3.

Mental Maths:

- To count forwards and backwards in 3s, 4s and 8s
- To count forwards and backwards in 3s, 4s and 8s from any given number
- To have rapid recall of all division facts when given a multiplication fact
- To use repeated subtraction on the counting stick (i.e. 18÷3= count back 15, 12, 9, 6, 3, 0 = 6)
- To divide any number by one or zero
  - To divide any two-digit even number by 2. Use known facts to derive quickly: doubles of numbers 1-100
  - doubles of 5,15, 25 up to 100
  - doubles of 50, 100, 150, 200 up to 500 and all corresponding halves
- To divide any three-digit multiple of 10 by 10 (340÷10, 890÷10 etc.)
- To know how to find quarters of a number by finding half of a half (quarter of 60 = 60÷2= 30÷2=15)

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping including knowledge of fractions	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems.  	How many groups of 6 in 24?  24 ÷ 6 = 4 I could use my knowledge of tables to do:  60 ÷ 3= 20 (because if I know 3x2= 6 then 3x20 = 60 therefor 60÷ 3= 20) 9 ÷ 3= 3 (I know that 3x3 = 9) 20 + 3 = 23 4 cakes shared between 8 children equals ½ each.

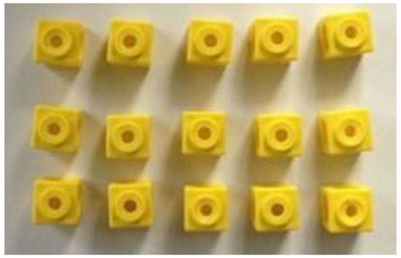
Link division to multiplication

Use the distributive law to divide "friendly" numbers.

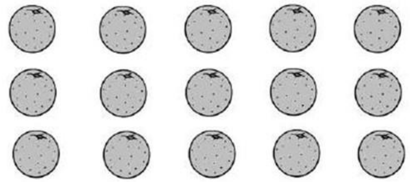
Link division to multiplication

E.g. 15 ÷ 3 = 5   5 x 3 = 15

15 ÷ 5 = 3   3 x 5 = 15



Draw an array and use lines to split the array into groups to make multiplication and division sentences

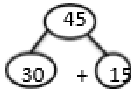


Find the inverse of multiplication and division sentences by creating eight linking number sentences.

7 x 4 = 28	28 = 7 x 4
4 x 7 = 28	28 = 4 x 7
28 ÷ 7 = 4	4 = 28 ÷ 7
28 ÷ 4 = 7	7 = 28 ÷ 4

45 ÷ 3= □

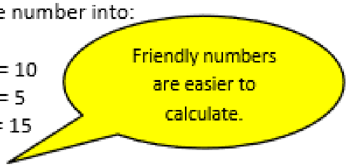
Use knowledge of 45 to split the number into:



30÷ 3= 10

15÷ 3= 5

10+5 = 15

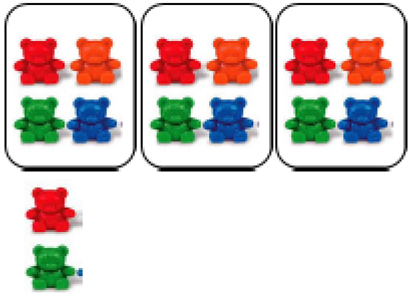


Therefor 45÷ 3 = 15

Division with remainders.

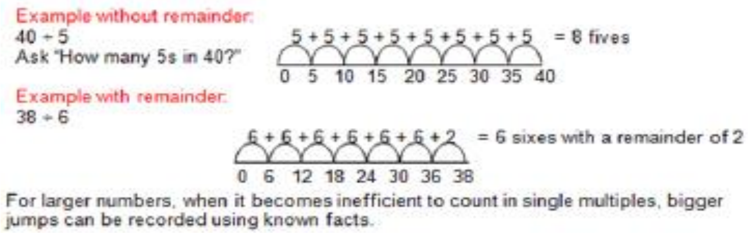
4 ÷ 3 =

Divide objects between groups and see how much is left over

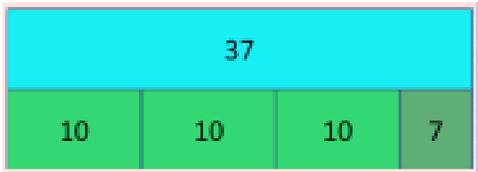


Draw dots and group them to divide an amount and clearly show a remainder.

Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder



Use bar models to show division with remainders.



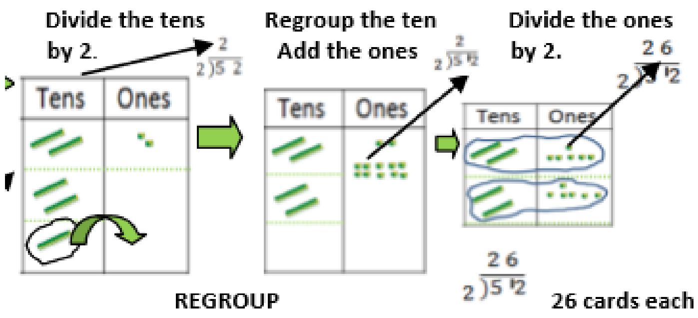
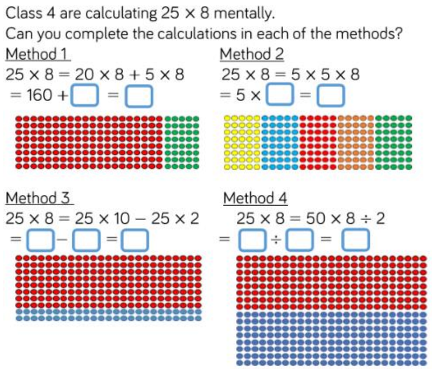
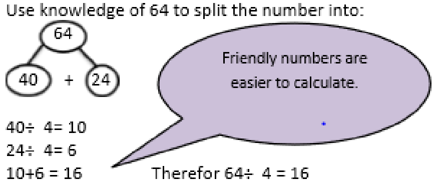
Complete written divisions and show the remainder using r.

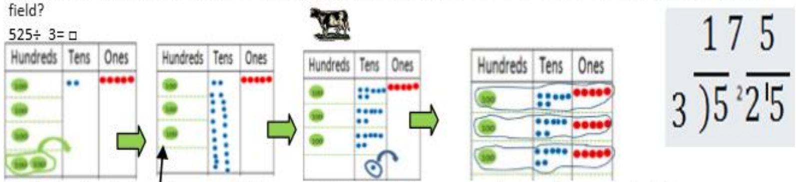
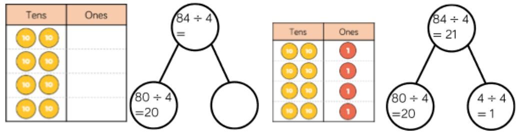
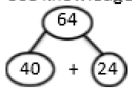
38÷6 = 6r2

Year 4 - Division (track back to year 3 and forwards to year 5 for progression)

National Curriculum:	**Potential barriers / misconceptions:**	**Mental Maths:**

• Recall multiplication and division facts for multiplication tables up to 12 × 12
• Use place value, known and derived facts to multiply and divide mentally, including:
  ○ multiplying by 0 and 1;
  ○ dividing by 1;
  ○ multiplying together three numbers
• Recognise and use factor pairs and commutativity in mental calculations
• Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
• Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
	• Pupils do not understand that division is grouping as well as sharing.
	• Pupils are muddled about the correspondence between multiplication and division facts, recording, for example: 3 X 5 = 15 so 3 ÷ 15 = 5.
	• Pupils write a remainder that is larger than the devisor, for example: 36 ÷ 7 = 4 remainder 8.
	• Pupils discard the remainder as do not understand its significance
	• Pupils continue to subtract two when calculating 20 divided by 2 without using knowledge that 2 multiplied by 5 equals 10.
		Rapid recall of multiplication facts to 12x12
		To know all related division facts when given a multiplication fact (8x4 = 32 therefor 32÷4 = 8 32÷8 = 4)
		Recognise and use factor pairs
		To give statements about odd and even numbers (An odd digit cannot be divided exactly by two)
		To know the divisibility of numbers (ring the numbers that divide exactly by four: 3, 8, 20, 27, 34, 36, 48, 50)
		Recognise that a whole number is divisible by: 100 if the last two digits are 00; 10 if the last digit is 0; 2 if the last digit is 0,2,4,6,8; 4 if the last two digits are divisible by 4; 5 if the last digit is 5 or 0
		Find all the pairs of factors of any number to 100 (i.e. pairs of factors of 24 are: 1 and 24, 2 and 12, 3 and 8, 4 and 6)
		Relate division to fractions (½ of 10 is the same as 10÷2 and ¼ of 12 is the same as 12÷4)
		To divide a whole number of pounds by 2, 4, 5 or 10 (£29 divided between 4 people = £7 each + £1÷4= 25p = £7.25 each)
		Understand halving as the inverse of doubling (*if double 37 is 74 then half 74 is 37*)
		To use related facts to half (i.e. half of 28 = half of 20 is 10 and half of 8 is 4 = 10+4=14)
		To find quarters and eighths by halving (1/8 of 56 is the same as half of 56 = 28 half again is 14, half again is 7 = 7)
		To divide a four digit multiple of 1000 by 10 or 100 (8000÷ 100 = 80) (To find one tenth, one hundredth etc.)

Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Use the distributive law to divide 2 digit and 3 digit numbers by 1 digit without remainders	52÷2 		64 ÷ 4= □ Use knowledge of 64 to split the number into:  40÷ 4= 10 24÷ 4= 6 10+6 = 16 Therefor 64÷ 4 = 16

	There are 525 cows on the farm. They need to be put into three different fields. How many cows will there be in each field? 525÷ 3= □  **Divide the hundreds By the 3.** 3)5 2 5 **Regroup = 22 tens** 3)5 ²2 5 **Divide tens by 3** 1 7 3)5 ²2 ¹5 **Regroup the remainder 10 Divide ones by 3.** 1 7 5 3)5 ²2 ¹5	Jack is dividing 84 by 4 using place value counters.  First, he divides the tens. Then, he divides the ones.  	64 ÷ 4= □ Use knowledge of 64 to split the number into:  Friendly numbers are easier to calculate. 40÷ 4= 10 24÷ 4= 6 10+6 = 16 Therefor 64÷ 4 = 16
To make links between arrays and short division	120 ÷ 3 = 40  	 	$\begin{array}{r} 40 \\ 3\overline{)120} \end{array}$ 12 ÷ 3 = 4 3 into 0 = 0 12 tens ÷ 3 = 40

Year 5 – Division (track back to Year 4 and forward to Year 6 for progression)

National Curriculum:

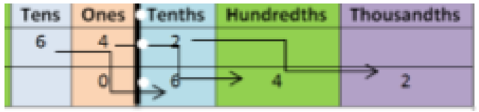
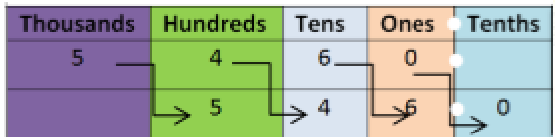
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply and divide numbers mentally drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Recognise and use square numbers and cube numbers, and the notation for squared(2) and cubed (3)
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Potential barriers/ misconceptions:

- Pupils recognise what calculation to do when word problems include the words 'times' or 'share', but are less confident when other language is used such as 'product', 'divided by', 'remainder' and mistakenly associate 'how many?' and 'how much?' with addition or subtraction.
- Understand multiplication as repeated addition and division as repeated subtraction but not as scaling up and down to prepare the way for later work in measures and on ratio.
- Pupils do not understand that ÷ 10 and then ÷ 10 again is the same as ÷ 100 When dealing with remainders, pupils have little understanding of how to represent as a fraction or a decimal.

Mental Maths:

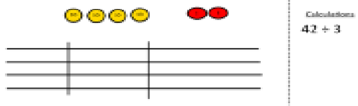
- To identify all factor pairs of a number
- To identify common factors of two numbers
- To recall prime numbers up to 19 To establish whether a number up to 100 is prime
- Multiply and divide numbers mentally using known facts. (i.e. *240÷3= 80 because I know 24÷3=8*)
- To use and understand the terms **factor, multiple and prime**, **square and cube** numbers
- To know that dividing by four is the same as finding a quarter etc. (and
- 1/3 of 24 is 24÷3)
- To divide any number by 10,100, 1000 To round up or down according to context (see year 3 exemplification)
- To double all whole numbers and decimals knowing that halving is the inverse
- Find sixths by halving thirds and twentieths by halving tenths

Objective & Strategy	Concrete	Pictorial	Abstract
Divide whole numbers and decimals by 10, 100 and 1,000	 **64.2 ÷ 100 = 0.642** (The place value shifts two places to the right and The value gets one hundred times smaller)  **5460 ÷ 10 = 546** (The place value shifts to the right and The value gets ten times smaller)	1÷10 = 1/10 or 0.1  Dividing by ten, you move the number one place value to the left.	64.2÷100=0.642

Divide at least 3 digit numbers by 1 digit.	Short Division			

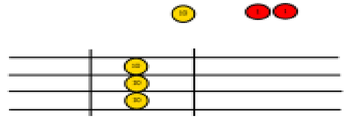
96 ÷ 3 Tens Units

Use place value counters to divide using the bus stop method alongside

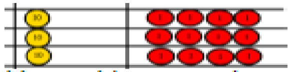


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.

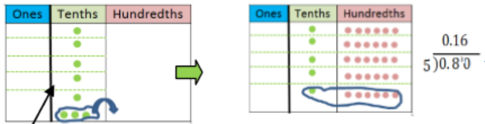


We look how much in 1 group so the answer is 14.

Decimals

A piece of ribbon is 0.8m long. Five children want part of the ribbon each for their hair. If the ribbon is shared equally how long is each piece?

$0.8 \div 5 = 0.16$



Each piece of Ribbon is 0.16m or 16cm.

Divide the ones by 5 hundredths

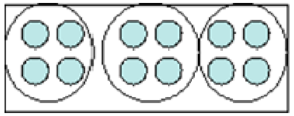
Divide the tenths by 5

= 1 tenth r 3tenths (regroup the 3)

3 tenths = 30

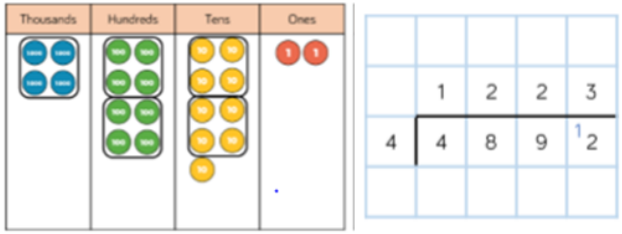
Divide by 5 = 6 hundredths

As we are dividing by five place value table is divided into five parts.



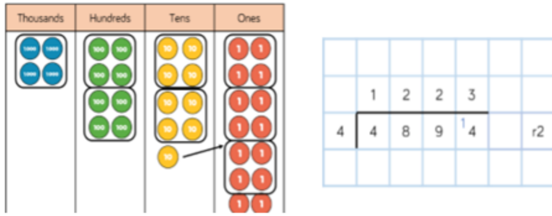
Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.

Encourage them to move towards counting in multiples to divide more efficiently.



Division with remainders

Here is a method to solve 4,894 divided by 4 using place value counters and short division.



Begin with divisions that divide equally with no remainder.

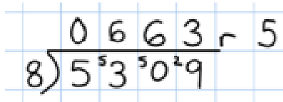
```
   2  1  8
       3
4| 8  7  2
```

Move onto divisions with a remainder.

```
   8  6  r 2
      3
5| 4  3  2
```

Finally move into decimal places to divide the total accurately.

```
      1  4 .  6
        16   21
3 5 | 5  1  1 .  0
```



$0.8 \div 5 = 0.16$

```
   0.16
5 )0.8'0
```

Year 6 – Division (Track back to Year 5 for progression) - *See Year 5 for short division*

National Curriculum:

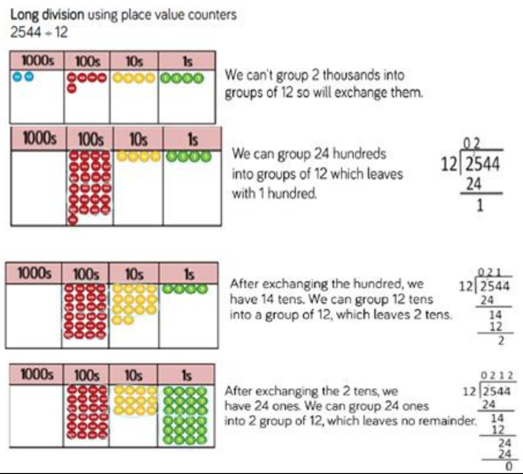
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Perform mental calculations, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve problems involving addition, subtraction, multiplication and division.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Potential barriers/ misconceptions:

- Pupils lack understanding that division is grouping as well as sharing
- Lack of times tables knowledge.
- Pupils ignore decimal point when calculating then ‘slot back in’ – due to generalisation of adding decimals.
- Pupils misunderstand the concept of making a no. 10/100/1000 times smaller, prefer to learn ‘knock off a zero’ and when a number ends in a different digit simply know that off.
- Ignore decimal point or ‘move it’.
- Pupils have a limited range of multiplication and division facts.
- Pupils misuse half understood rules about multiplying and dividing by powers of ten and the associative law, for example: 145 X 30 = 145 000
- Pupils have difficulty interpreting, when appropriate, a remainder as a fraction, for example: $16 \div 3 = 5\frac{1}{3}$
- Pupils interpret division only as sharing and not grouping (repeated subtraction) so are unable to interpret calculations such as: $12 \div \frac{1}{2}$.
- Pupils may not be confident in making reasonable estimates for multiplication or division calculations.

Mental Maths:

- Identify common factors & multiples
- To identify prime numbers
- Rapid recall of all multiplication tables (and related number families)
- To divide any number by 10, 100, 1000 (knowing that the place value changes)
- To find one hundredth or one thousandth of an amount by dividing by 100 or 1000
- To relate fractions to division (dividing by the denominator)
- To know doubles of numbers including decimals and corresponding halves
- To recognise that if 5x60 = 300 than of 300=60 and of 300 = 50
- To halve a decimal fraction less than 1 with one or two decimal places (half of 0.7)
- Use knowledge that in exact multiples of (and prove): 100 the last two digits are 00 and 10 the last digit is zero and 5
- The last digit is 0 or 5 25 The last two digits are 00, 25, 50 or 75 2
- The last digit is 0,2,4,5,8, 3
- The sum of the digits is divisible by 3 4
- The last two digits are divisible by 4 6
- The number is even and divisible by 3. 8
- The last 3 digits are divisible by 8 9
- The sum of the digits is divisible by 9.

Objective & Strategy	Concrete	Pictorial	Abstract
Long Division 3 digit divided by 2 digit Expanded method (using multiples to help)	see grid below		432÷12= Bus stop method

Concrete:

		3	6	
12	4	3	2	
–	3	6	0	(×10)
		7	2	
–		7	2	(×6)
			0	

Multiples to help
12 × 1 = 12
12 × 2 = 24
12 × 5 = 60
12 × 10 = 120

Long Division
4 digit divided by 2 digit

(subtracting from the total – seeing division as repeated subtraction)

Here is a division method.

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400)
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

7335 ÷15 =

489

15 | 7335

- 60

133

- 120

13 5

Long Division with remainders

Elijah uses this method to calculate 372 divided by 15
He has used his knowledge of multiples to help.

		2	4	r	12
15	3	7	2		
–	3	0	0		
		7	2		
–		6	0		
		1	2		

1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150

Interpret remainders appropriately to the context.

Problem	Appropriate remainder
113 litres of water poured equally into 4 ten litre containers. How much water in each?	113÷4 = 28.25 = 28.25 litres
113 children in 4 equal groups. How many groups are formed? How many are not in a group?	113÷4 = 28 r1 28 groups and 1 child not in a group.
Taxis that hold 4 people are used to drive 113 people home. How many taxis are needed for everyone to get home?	113÷4 = 28 r1 29 taxis are needed
The class put 113 cookies into boxes of 4. How many were left over to eat?	113÷4 = 28 r1 1 cookie to eat
113 cookies in boxes of 4. How many boxes are needed? What fraction of the incomplete box is filled?	113÷4 = 28 $\frac{1}{4}$ 28 boxes are needed The last is $\frac{1}{4}$ filled.

Year 6 Long Division

Step 1—a remainder in the ones

```
     h t o
     0 4 1 R1
4 )  1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th h t o
     0  4 0 0 R7
8 )  3  2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Step 2—a remainder in the tens

```
     h t o
     0 6 1
4 )  2 4 7
       - 4
         3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
     th h t o
     0  4 0 2
4 )  1  6 0 9
          - 8
            1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8 Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	t o 2 2) 5 8 - 4 1 To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	t o 2 9 2) 5 8 - 4 ↓ 1 8 Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8 Divide 2 into 18. Place 9 into the quotient.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0 The division is over since there are no more digits in the dividend. The quotient is 29.

Year 6 Long Division

Step 3—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2) 2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2) 2 7 8 - 2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2) 2 7 8 - 2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2) 2 7 8 - 2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0 There are no more digits to drop down. The quotient is 139.